

## PHILIP ORDING

Philip Ording is a writer and member of the mathematics faculty at Sarah Lawrence College. His first book, *99 Variations on a Proof*, also published by Princeton University Press, received the 2020 PROSE Award for Excellence in Physical Sciences & Mathematics.

Troels's office was on the sixth floor: one flight up the oversize McKim, Mead & White staircase from the math lounge, which boasts the board pictured here; two flights up from what would eventually be my grad-student cubicle; and three flights up from the library circulation desk I then staffed as an undergrad work-study student.

I didn't know what his name meant in the field of geometry when he agreed to supervise my senior thesis. To me, he was a tall, friendly, unassuming Dane. And an extraordinarily patient professor. My answers to his questions met with a "Well, let's see." He would then stand up from his oversized office chair, take a piece of chalk, and draw a figure or write down a Möbius transformation in what seemed like a single gesture. The confidence of these marks on the otherwise spotless board (there didn't even seem to be any dust in the chalk tray) compelled belief in me. Understanding took longer. At one point, rather than give an uncertain answer to a question, I just sat staring at the board. He watched me.

"My mother made me take piano lessons when I was a boy," he finally said. "In the first lesson the teacher asked, 'What do you notice about the keyboard?'" I looked at Troels. "After a very long pause, maybe thirty seconds, the teacher gently remarked, 'Some of the keys are white and some are black.'"

This was Troels's way. If something was obvious to him, he didn't expect we'd feel that way. He knew when we were stuck but was never bothered by it and never showed any disappointment. He identified with his students.

By spring I had begun to make enough progress to take a turn at the board. It was oddly empowering. The surface was perfectly smooth except in an upper corner where a

hairline crack was visible at close proximity. Compared to the sliding, enameled porcelain chalkboards in the lecture halls, the chalk produced an inexplicably even sound on this board. And its surface didn't give like the cheap painted Masonite green boards in school. Troels's blackboard was made of slate.

Ever indulgent, he explained how not to make a mess of one's board work. How to keep chalk from squeaking. How to draw figures. The trick to drawing a round circle is posture and momentum. You have to face the board directly—even if it means turning your back to an audience—wind up at the elbow, and do it in one go. To draw a sphere, add two poles and the equator. But don't put the poles on the circle; they never appear simultaneously, except from a point in the equatorial plane, "at infinity." Do draw the equator as though the sphere were partially transparent, with a dashed line for the back half if you want. But don't give it corners like a lens. In the picture plane of the blackboard, the equator projects to an ellipse.

The board in this photo shows how not to draw a sphere.

Of course no one, including Troels, really cares how artful one's board work is if the math isn't interesting. (Was drawing ever a prerequisite for being a geometer?) But a diagram does give us another way to think—or, at least, something to stare at—when language fails. And figuring out for oneself what is and isn't mathematically interesting is never black and white.

