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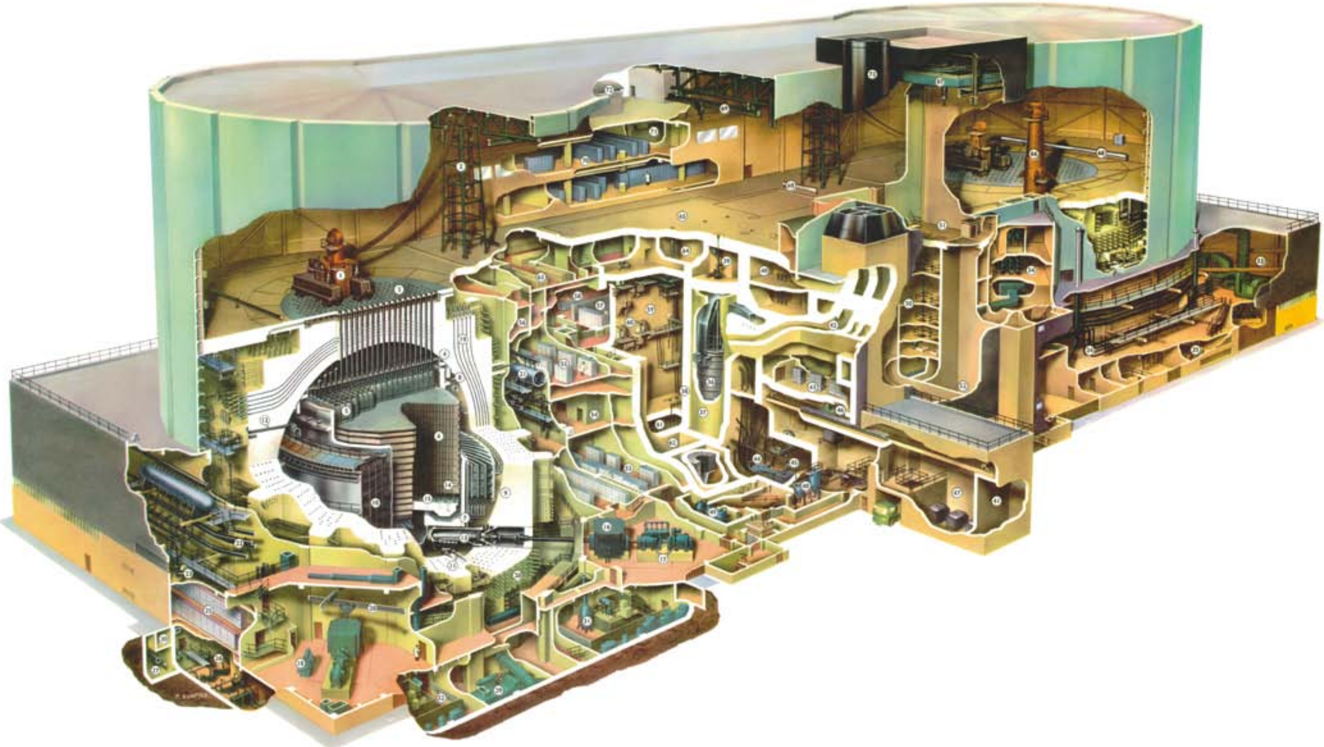


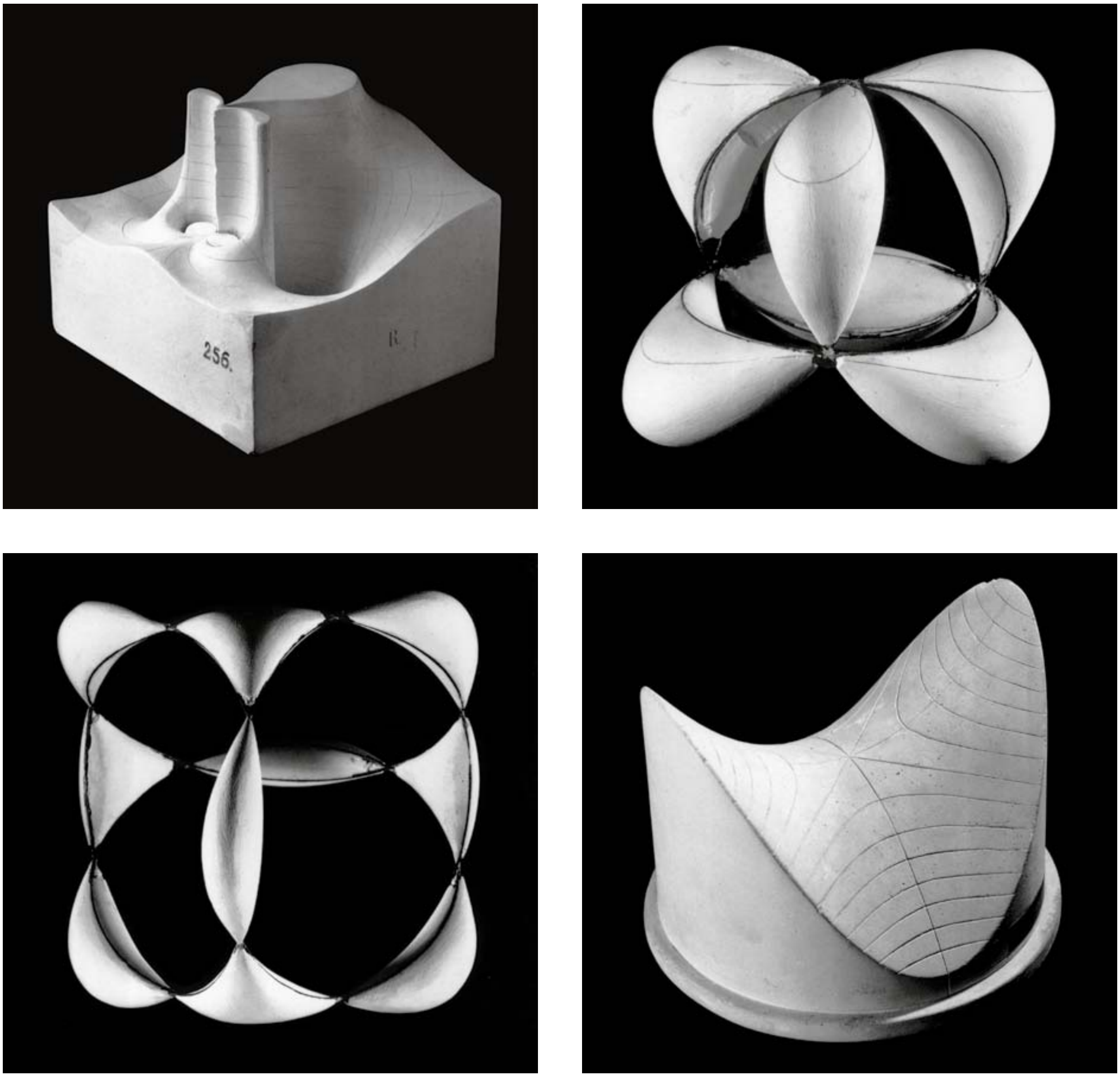
Man Ray, *Mathematical Object*, 1936.

MATHEMATICAL OBJECTS

PHILIP ORDING

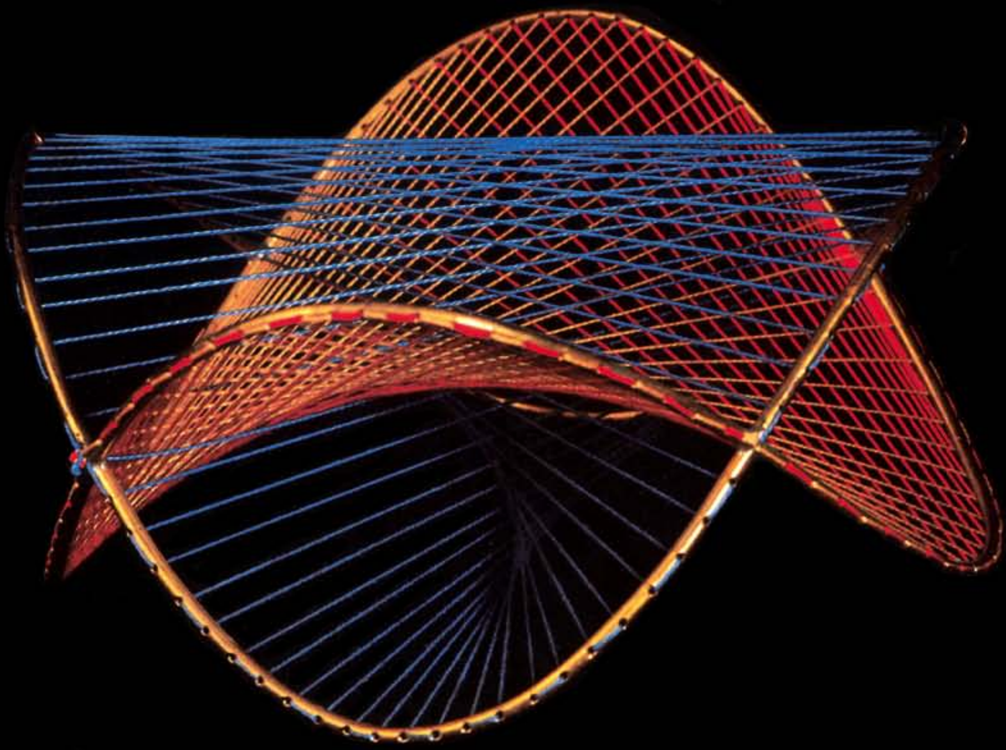
There was a moment in the history of mathematics when the construction of physical representations of mathematical concepts was an integral part of research. In the period from 1870 to 1930, most of the leading mathematical institutions in Europe and North America collected numerous mathematical models made from a variety of materials including plaster, metal, paper, wood, and string. Germany was the primary source of mathematical models; at the Munich Technical College, Felix Klein and Alexander Brill established a laboratory for the design, production, and pedagogical application of models. Their doctoral students made models in connection with their dissertations, and the third annual meeting of the German Mathematicians' Association, held in 1893, included an extensive exhibition of mathematical models. Ludwig Brill, Alexander's brother, had a bookshop in Darmstadt, and he began to sell the models in series with an accompanying mathematical guide. Mathematical models were also produced elsewhere in Europe—probably first in France at the École Polytechnique—but the German publishers were the source of most of the models to be found in mathematical institutes around the world. As the official representative of the Prussian Government to the World Columbian Exposition of 1893, Klein—best known today less for these models than for the Klein Bottle, a closed surface with no inside or outside—brought a large selection of the mathematical models in production for the exhibits in Chicago. In 1899, Martin Schilling took over publication of the models from Brill, and his 1911 catalogue advertised over four hundred models for sale. What do these objects reveal about how mathematicians *see* mathematics?

"It seems difficult to develop a correct imagination of this second version of Boy's surface without having a model at hand,"[1] Gerd Fischer writes in the commentary accompanying his comprehensive photographic volume of mathematical models. The mathematical object in question is a particular conformation of the



above (clockwise from top left): Mathematical models showing graph of $6w=e^{1/6z}$; quartic with four D_4 double points; hyperbolic paraboloid with plane sections; quartic with twelve A_1 double points.

opposite: Model of hyperbolic paraboloid and cylindroid. All images on this and following page from Gerd Fischer, *Mathematical Models*, 1986.



topological surface obtained by joining the edge of a Möbius band to a disk, but he could be speaking about nearly any model from the period. The mathematical study of surfaces and curves occupied a significant portion of mathematical research during this model-making epoch, and many of the fine plaster models, such as the Kummer surface, depict interactions between two-dimensional spaces arising from algebraic equations (the conic sections are a one-dimensional example of the same idea). The graphs of functions of complex variables also inspired several plaster models. Stringed constructions form a large portion of the models in university collections, though few of these delicate objects remain intact. In these models, each array of colored string traces out a spanning surface, known as a "ruled surface," which held important connections to the mathematics underlying both perspective painting and the topological surface described above.

That the models could help "to develop a correct imagination" seems to have been the rationale for their use in education. For Klein, at least during part of his career, models were more than pedagogical tools: "There is an essential geometry ... it is the task to grasp the spatial figures in their full figurative reality, and (which is the mathematical side) to understand the relations valid for them as evident consequences of the principles of spatial intuition. For this geometry a model—be it realized and observed or only vividly imagined—is not a means to an end but the thing itself."[2]

Klein was chair of mathematics at the University of Göttingen from 1886 until he retired in 1913, and his organizational and political skills are credited for making Göttingen the exemplary research center that it was at the time. The Mathematical Institute there, conceived by Klein but not fully realized until 1929, after his death, offers the most comprehensive collection of mathematical models on view anywhere in the world. Filling the large, sunny room between the library and lecture hall, the collection occupies several rows of tall, wooden cabinets with glass doors and translucent tops. The models seem to have played an interesting role in the development of mathematics as an academic discipline. "At the turn of the century mathematical institutes, as connected systems of rooms, were still rare, but every university had a more or less extensive collection of models ... Klein had paved the way. His own interest in models was deep, but in other cases the motive for the collection may just have been the quest for rooms and self-representation,"[3] writes historian of mathematics Herbert Mehrtens. Collecting things that required space for storage gave mathematicians—who otherwise needed nothing but a desk to do their work—a rationale for more real estate. Like a chemist's molecular model or an astronomer's orrery, the models asserted the existence of the mathematician. Ironically, the success with which models came to symbolize the profession probably contributed to their obsolescence.

Formalism would become the dominant mathematical attitude of the twentieth century, as mathematicians sought distance from the sciences in favor of what was viewed as a return to fundamentals and logical deduction. This purist trend (its currents are still forceful today) is most widely associated with the *Elements of Mathematics*, a series of texts written by a group of French mathematicians under the pseudonym Nicolas Bourbaki—diagrams and geometrical figures were completely absent from their influential volumes. Economic conditions, combined with the change in the mathematical climate, slowed the production of mathematical models. In 1932, the year Bourbaki's first volume, *Set Theory*, appeared, Schilling wrote to the institute in Göttingen that "in the last years no new models have appeared. ... [There] are various new models in preparation. However, as a result of the bad and unpredictable market conditions, we have held back their production." This was Schilling's last correspondence with Göttingen; he never published any additional models.

At this point, however, the models began to reach another audience as artists discovered the appeal of their strange forms. Most notably, Max Ernst incorporated images from Schilling's catalogue into his surrealist collages; sometime around 1935, he also brought his friend and fellow surrealist Man Ray to see the models in Paris. Man Ray recalled, "One day I was told about some mathematical objects at the Institut Poincaré in Paris. These were built by the [professors] to explain algebraic equations. I went to see them, although I am not particularly interested in mathematics. I didn't understand a thing, but the shapes were so unusual, as revolutionary as anything that is being done today in painting or in sculpture. And I spent several days photographing and sketching them with the intention of doing a series of paintings influenced and inspired by these objects."[4] Prior to these artistic uses of the image of the mathematical model, the Russian artist Naum Gabo, who studied engineering at the technical college in Munich, had drawn on the formal aspects of the models in his sculpture. And artists would continue to rediscover the models and find inspiration in their unexpected shapes and peculiarities. Charles and Ray Eames, for instance, installed a case of the models at the entrance to their

Apollo Belvedere showing off his parabolic curves in the Hilbert room at the University of Göttingen. Courtesy Samuel J. Patterson and the Mathematical Institute, Göttingen.

"Mathematica: A World of Numbers... and Beyond" exhibition, which has been on view in several American museums (including the Boston Science Museum, where I first encountered the surprising forms on a school field trip) since the IBM-commissioned exhibition first opened in 1961. As recently as 2004, the *New York Times* reproduced a series of photographs entitled "Mathematical Forms" by the photographer Hiroshi Sugimoto depicting Brill's plaster models, which he encountered at the University of Tokyo.

In Göttingen, prominent place is given to one of the most unexpected objects in the collection, one that merges aesthetics with mathematical speculation. On an upper shelf near the main entrance of the Hilbert room—which houses the models and is named for one of the institute's first directors and most influential mathematicians, David Hilbert—rests a large classical bust. The inscription on its base reads, "Apollo of Belvedere, with parabolic curves." Closer inspection reveals many thin black curves wandering over the surface of the right side of the figure's face and neck. In differential geometry, surface curvature is characterized as being either sphere-like or saddle-like (meaning that it curves in opposing directions), and parabolic curves distinguish regions of one type of curvature from those of the other. For example, a parabolic curve across Apollo's nose separates the tip from the bridge, because the tip is rounded, whereas the bridge is more like a saddle. Hilbert and his student Stephan Cohn-Vossen recount that Felix Klein "used the parabolic curves for a peculiar investigation. To test his hypothesis that the artistic beauty of a face was based on certain mathematical relations, he had all the parabolic curves marked out on the Apollo Belvedere, a statue renowned for the high degree of classical beauty portrayed in its features. But the curves did not possess a particularly simple form, nor did they follow any general law that could be discerned."[5]

What is discernable in the mask of parabolic curves is Klein's own mathematical eye. According to one of his students, Grace Chisholm Young, "His favorite maxim was 'Never be dull!'"[6] For Klein, the tension between formal and informal modes of mathematical activity was a productive force, not only at the level of the working mathematician—for me, his Apollo is a vivid example—but also on the level of the historical development of mathematics. In a lecture he delivered at Northwestern University on the occasion of his Chicago visit, Klein distinguished between what he called "the *naïve* and the *refined* intuition": "It is the latter that we find in Euclid; he carefully develops his system on the basis of well-formulated axioms, is fully conscious of the necessity of exact proofs ... and so forth. The naïve intuition, on the other hand, was especially active during the period of the genesis of the differential and integral calculus. Thus we see that Newton assumes without hesitation the existence, in every case, of a velocity in a moving point, without troubling himself with [exceptional cases]. At the present time we are wont to build up the infinitesimal calculus on a purely analytical basis, and this shows that we are living in a *critical* period similar to that of Euclid."[7]

1 Gerd Fischer, *Mathematical Models* (Braunschweig: Vieweg, 1986), p. 65.
2 Felix Klein, *Das Erlanger Programm: Vergleichende Betrachtungen über neuere geometrische Forschungen* (1872), ed. Hans Wussing (Leipzig: Akademische Verlagsanstalt, 1974), quoted in Herbert Mehrtens, "Mathematical Models," in *Models: The Third Dimension of Science* (Stanford: Stanford University, 2004), p. 289.
3 Mehrtens, op. cit., p. 284.
4 Edwin Mullins, *A Life in the Day of Man Ray* (Chicago: Homevision, 2000), VHS, quoted in Angela Vierling-Claassen, "Bridges Presentation" available at <angelavc.wordpress.com/2010/07/25/bridges-presentation> (accessed 31 March 2011).
5 David Hilbert and Stephan Cohn-Vossen, *Geometry and the Imagination*, trans. Peter Nemenyi (New York: Chelsea, 1952), p. 198. Originally published as *Anschauliche Geometrie* (Berlin: Springer-Verlag, 1932).
6 Grace Chisholm Young, "Obituary: Professor Klein," *The Times* [London], 9 July 1925.
7 Jeremy J. Gray and David E. Rowe, "Felix Klein at Evanston, Teaching and Doing Mathematics in 1893," *For the Learning of Mathematics*, vol. 13, no. 3 (November 1993), pp. 31–36.