

# An Example of Newton's Method

To calculate the **square root of 5** using Newton's method:

1. We write down a formula that equals zero when we plug in the value  $x = \sqrt{5}$ . That equation is:  $f(x) = x^2 - 5$
2. Next, we form the **derivative** of  $f(x)$ , which gives us  $f'(x) = 2x$   
(In general, the derivative of  $x^A - B$  for any  $A$  and  $B$  is just  $Ax^{A-1}$ )
3. Now we make a new formula from  $f$  and  $f'$ :

$$x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)} \quad \text{which, in this case, is:}$$

$$x_{t+1} = x_t - \frac{x_t^2 - 5}{2x_t}$$

# An Example of Newton's Method

Starting with any random value for  $x_0$  and iterating this equation, we will quickly converge to a very close approximation of  $\sqrt{5}$

$$x_{t+1} = x_t - \frac{x_t^2 - 5}{2x_t}$$

Current  $x$  value

Current  $x^2$  value

$$x = 1.00000$$

$$x^2 = 1.00000$$

$$x = 3.00000$$

$$x^2 = 9.00000$$

$$x = 2.33333$$

$$x^2 = 5.44444$$

$$x = 2.23810$$

$$x^2 = 5.00907$$

$$x = 2.23608$$

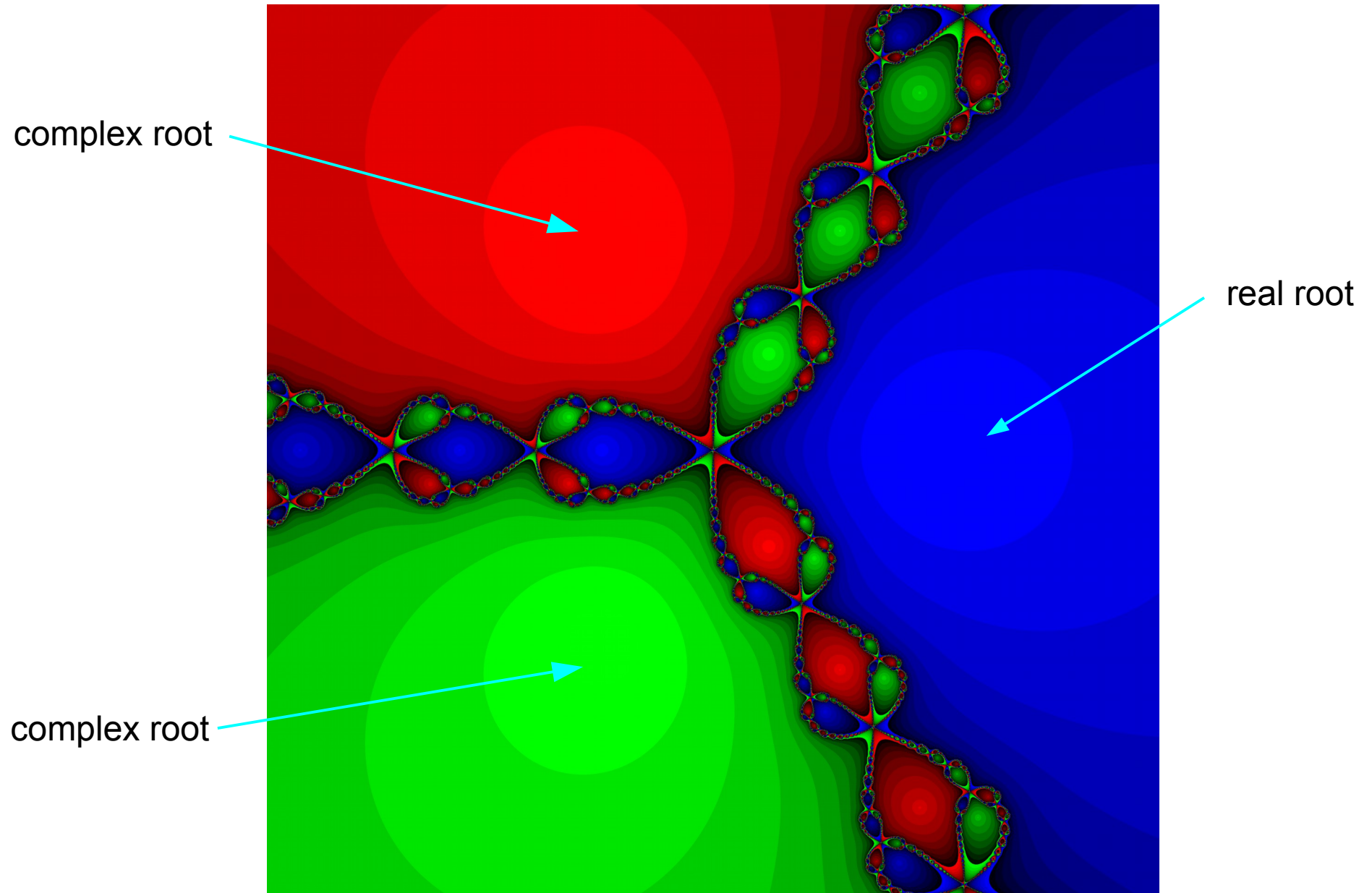
$$x^2 = 5.00005$$

$$x = 2.23607$$

$$x^2 = 5.00000$$

...converged in 5 steps

# Newton's Fractal (Cube Roots)



# Newton's Fractal (5th Roots)

