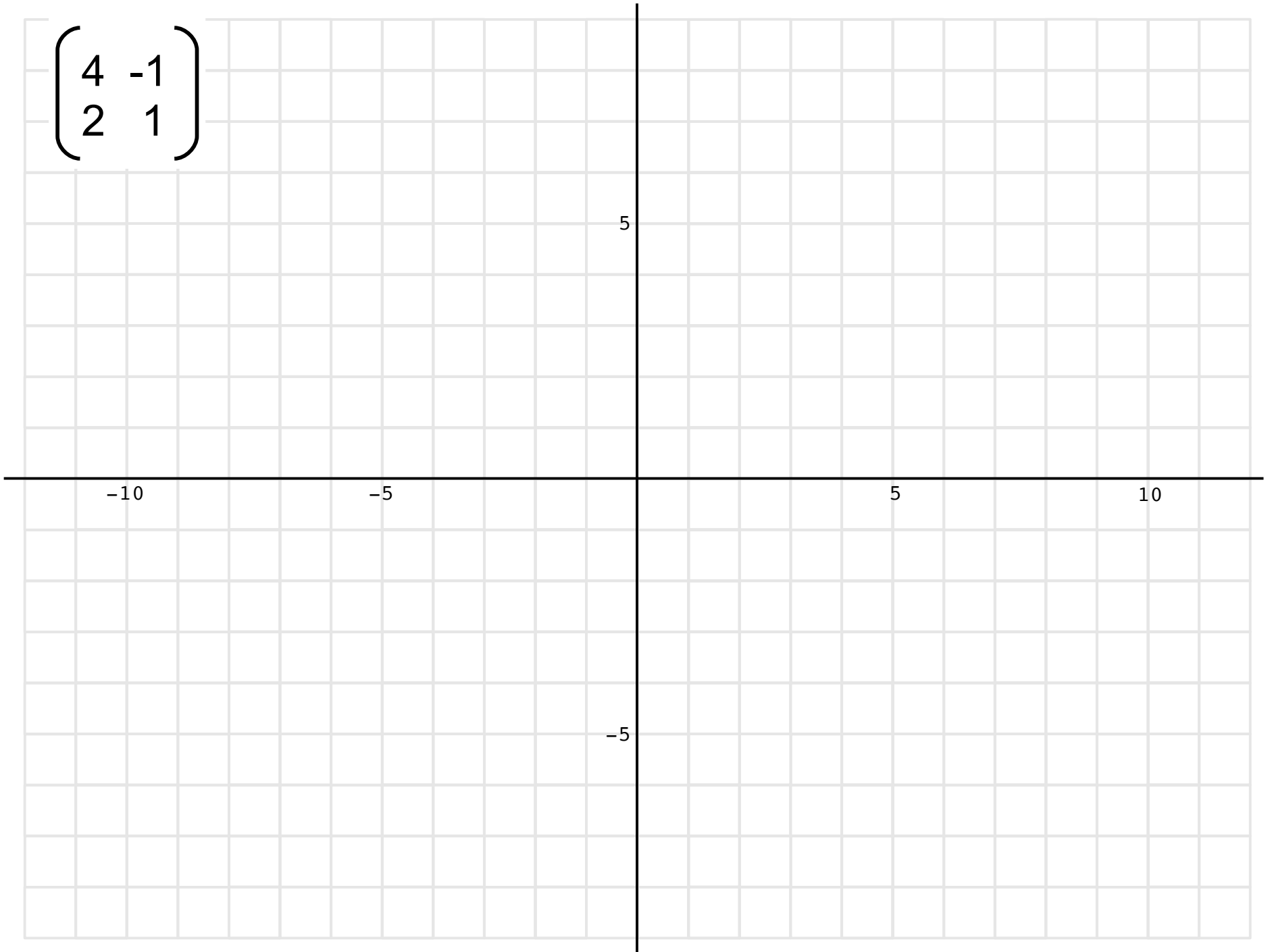
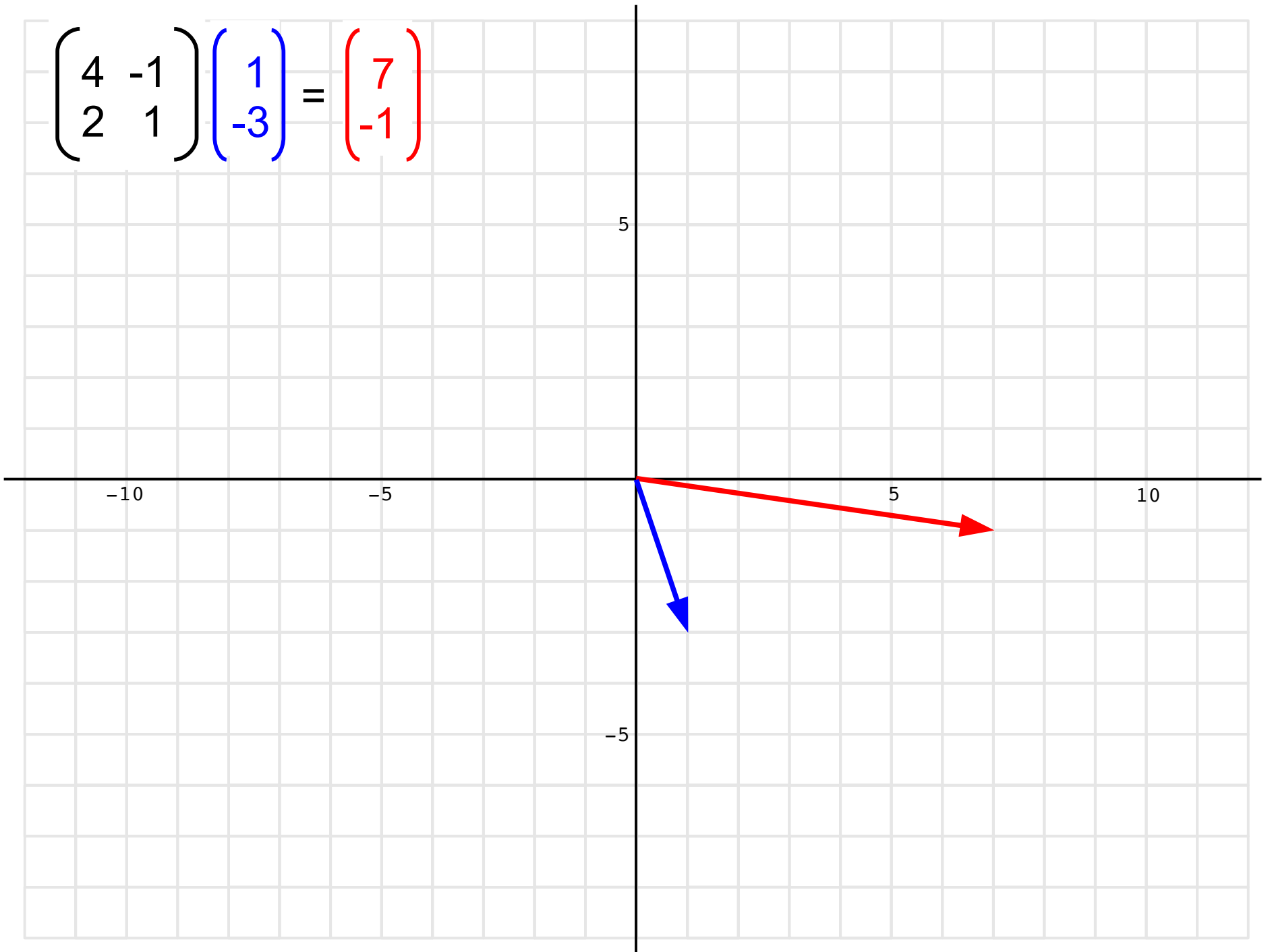


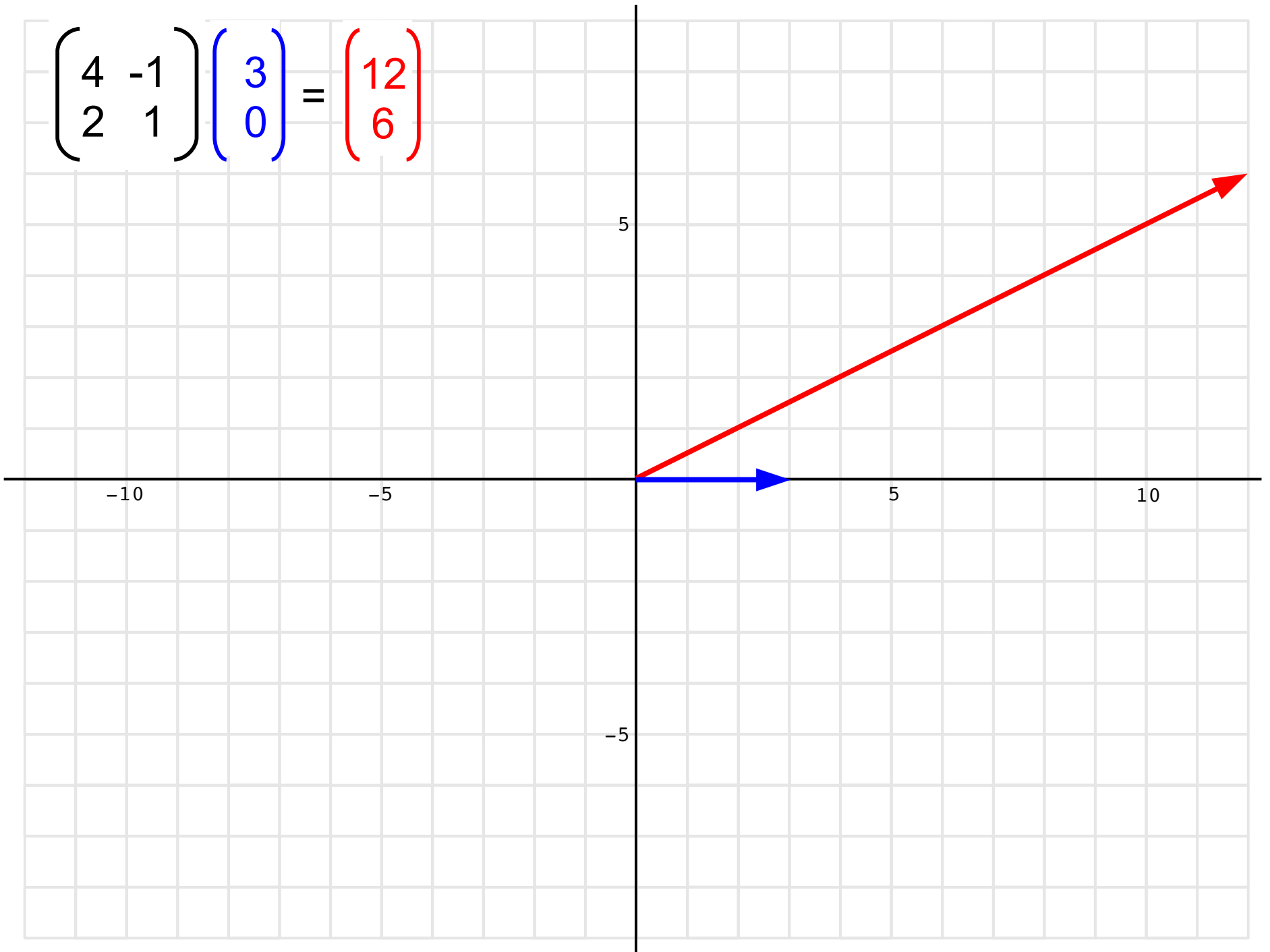
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$



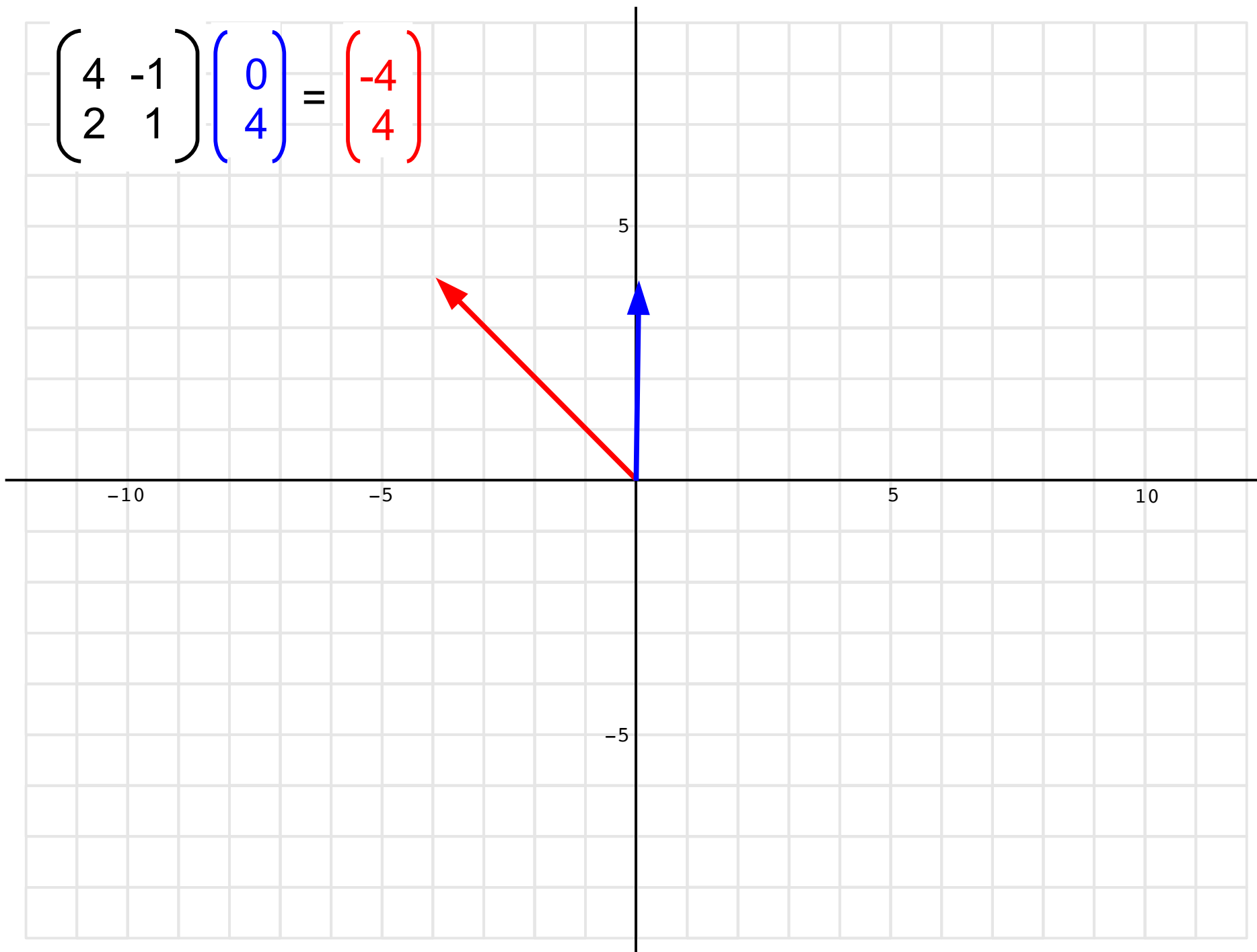
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$



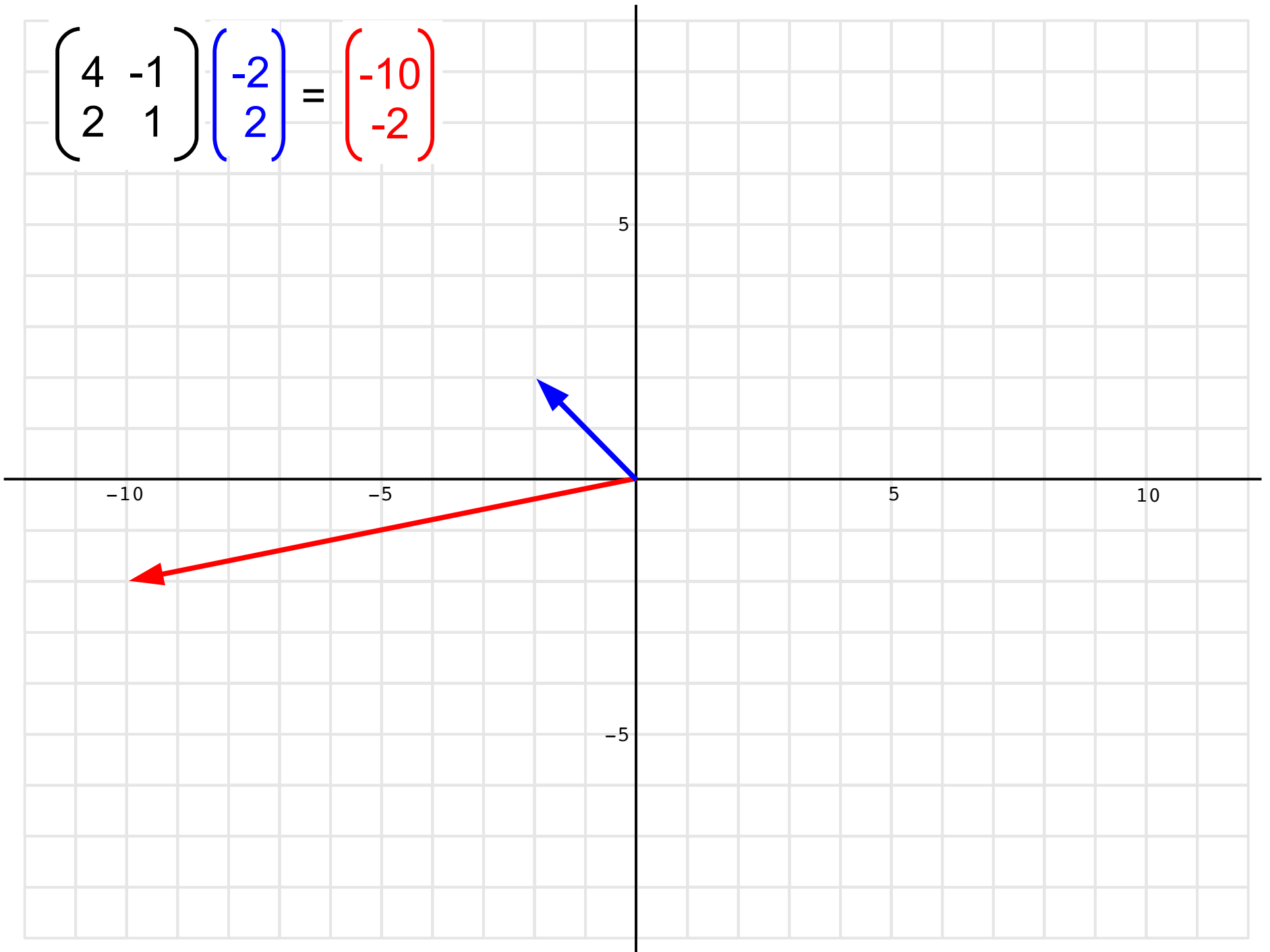
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$



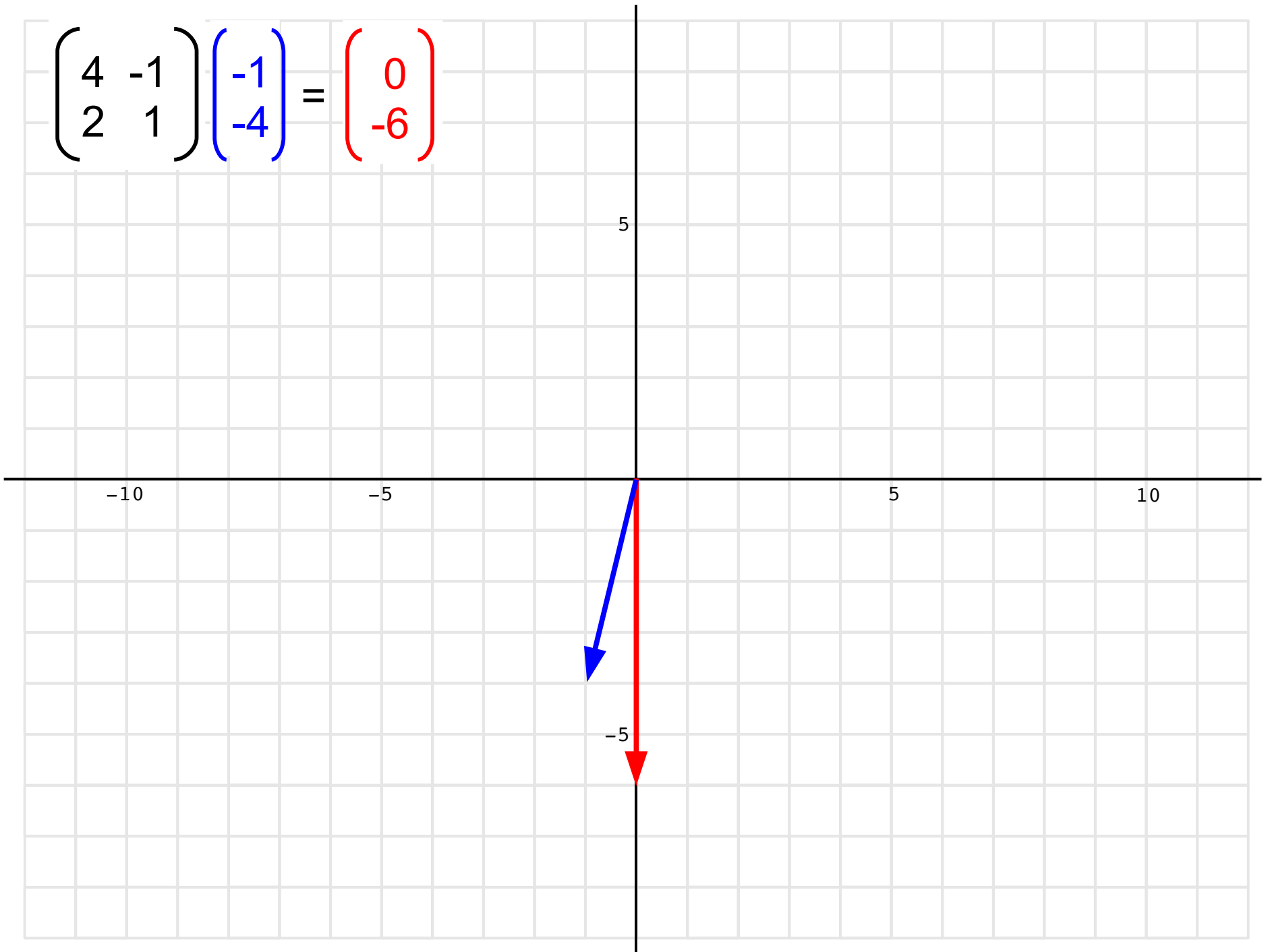
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$



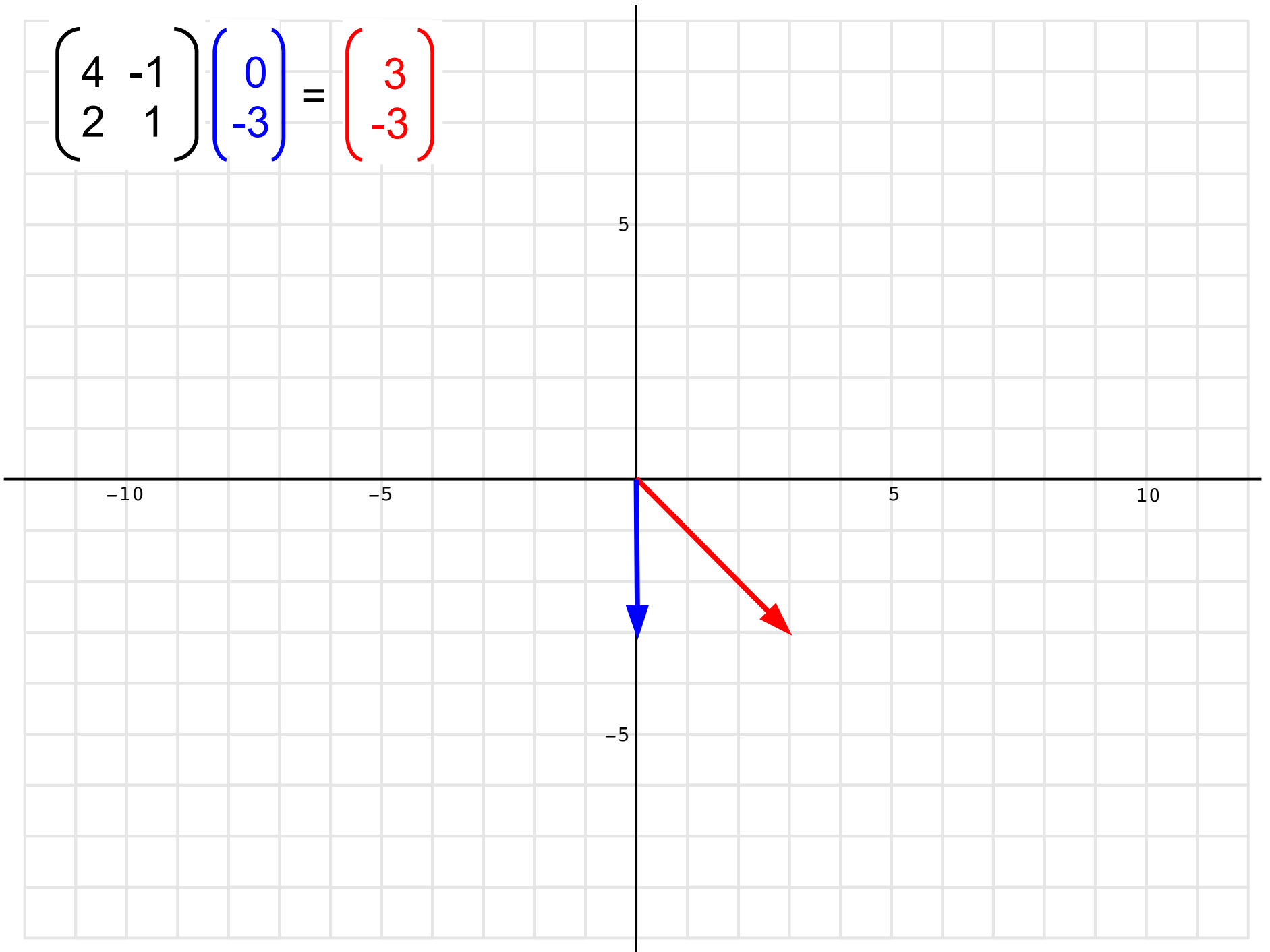
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -10 \\ -2 \end{pmatrix}$$



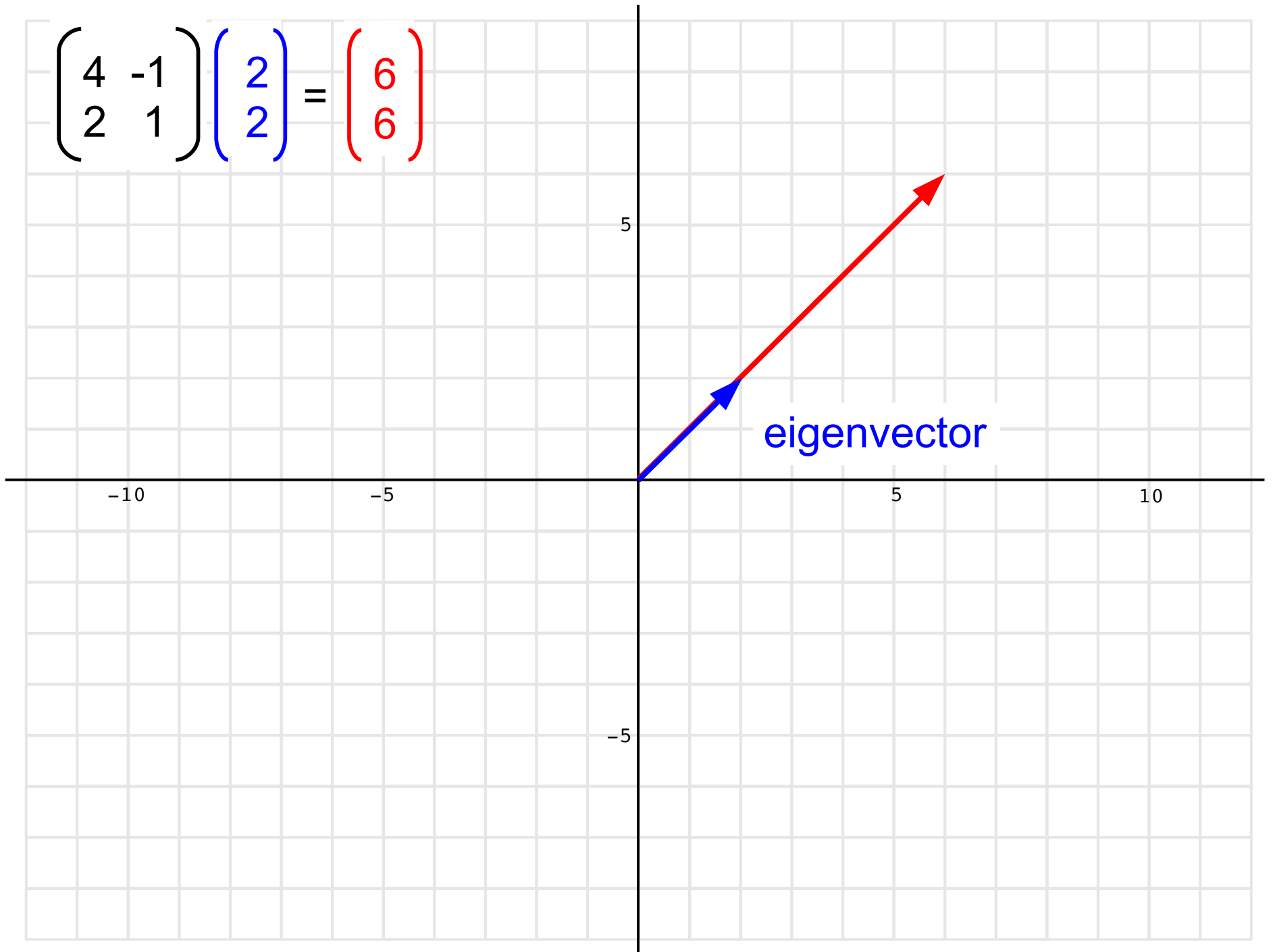
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$$



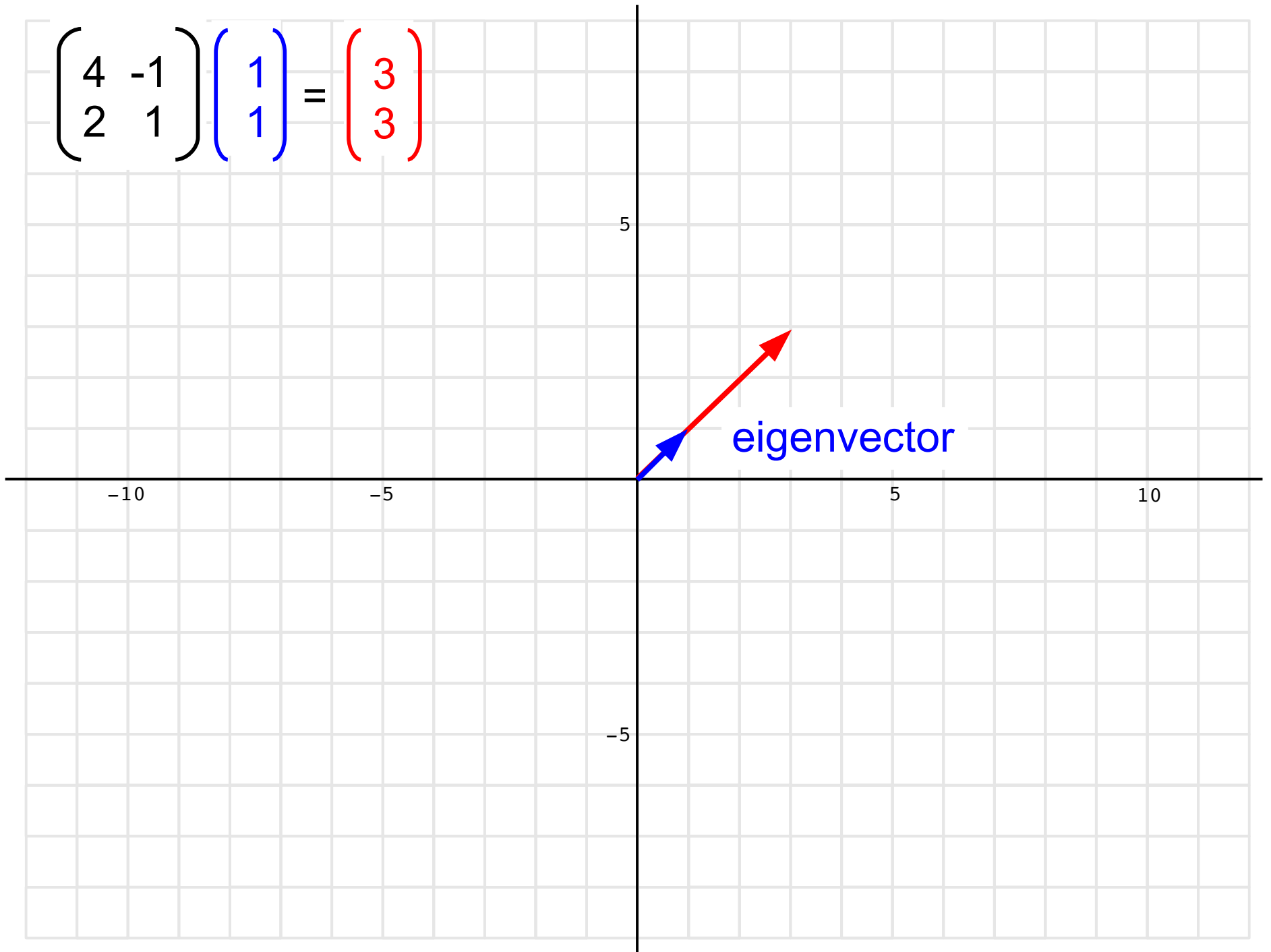
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$



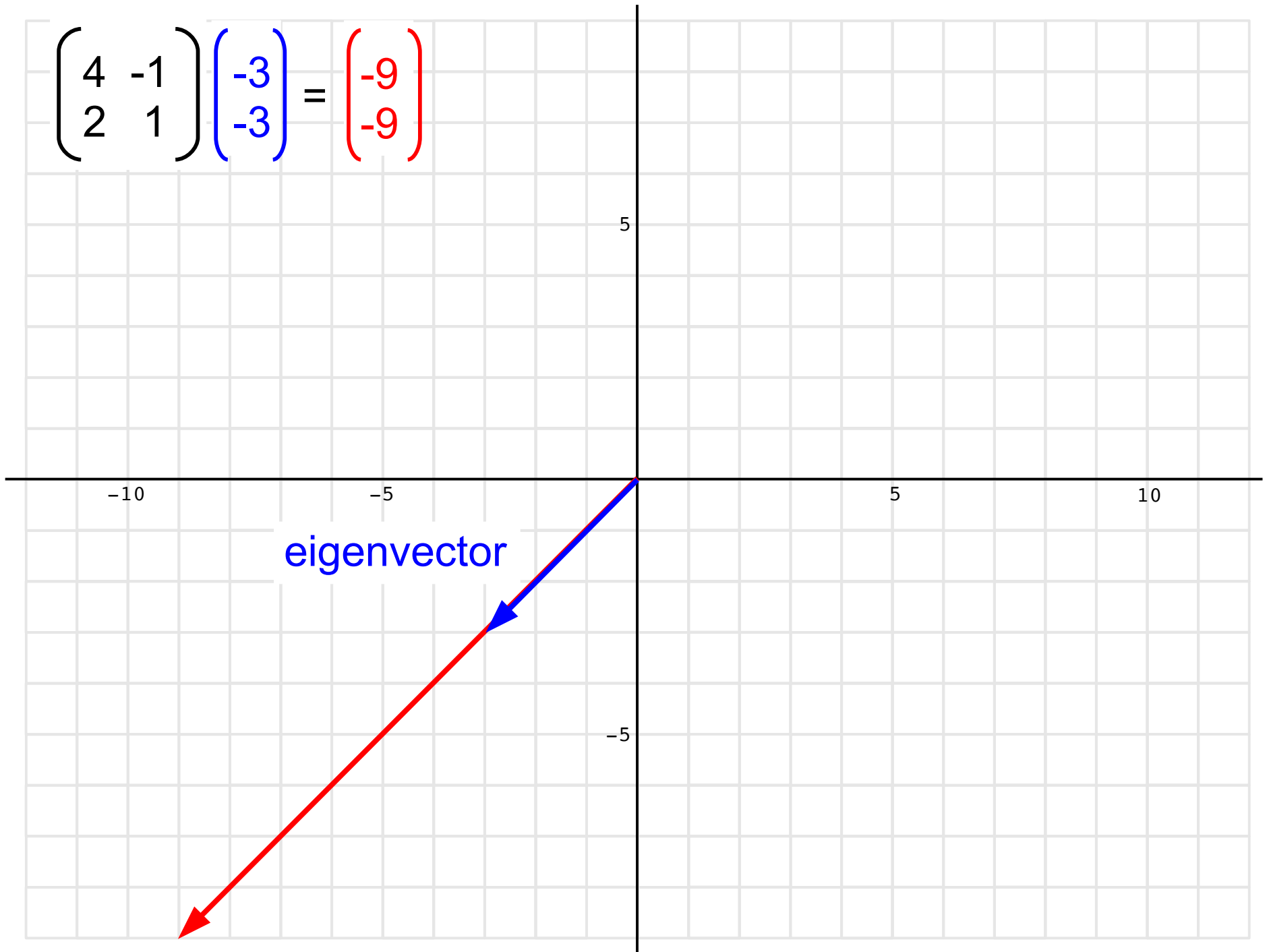
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$



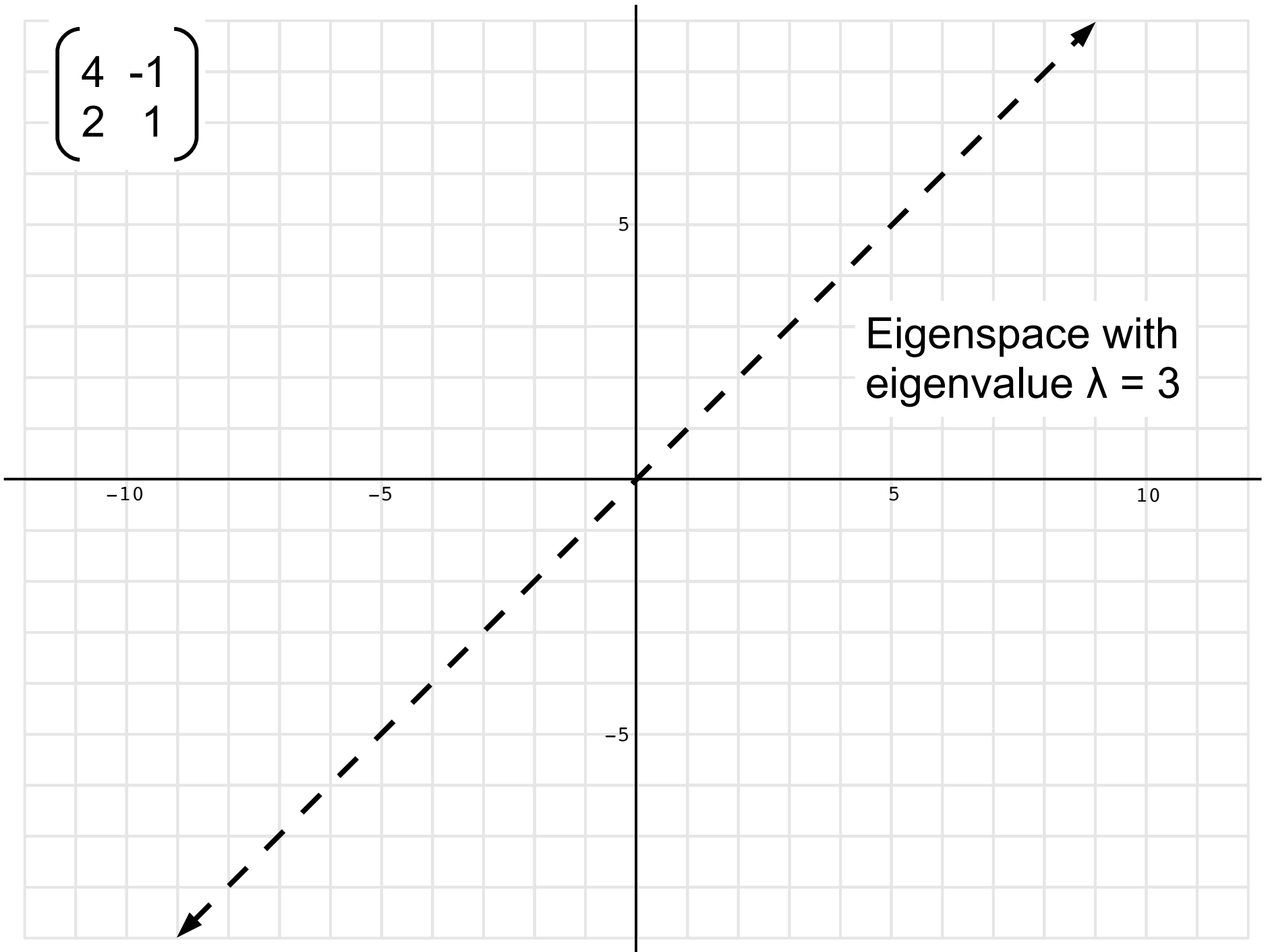
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$



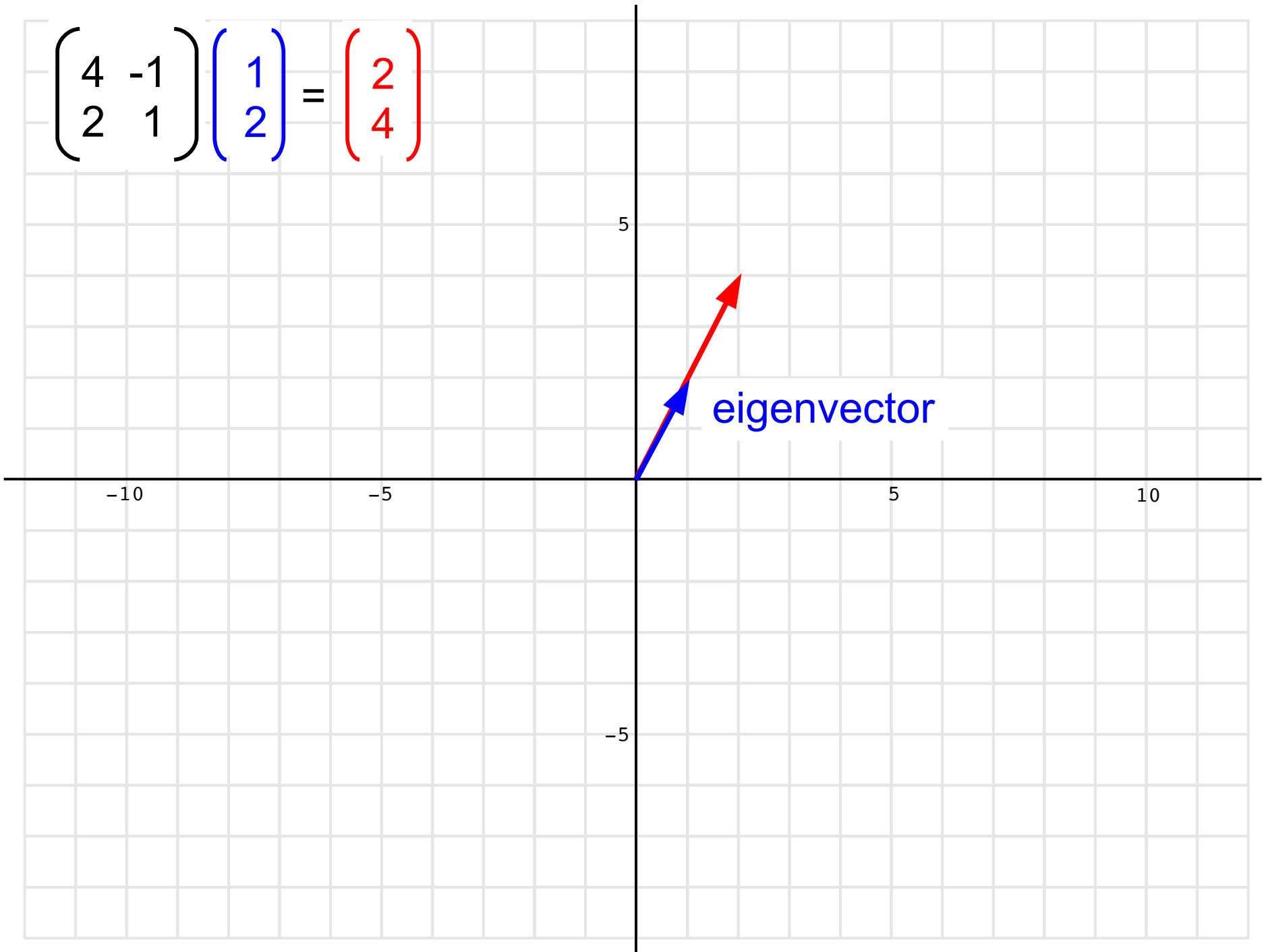
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ -3 \end{pmatrix} = \begin{pmatrix} -9 \\ -9 \end{pmatrix}$$



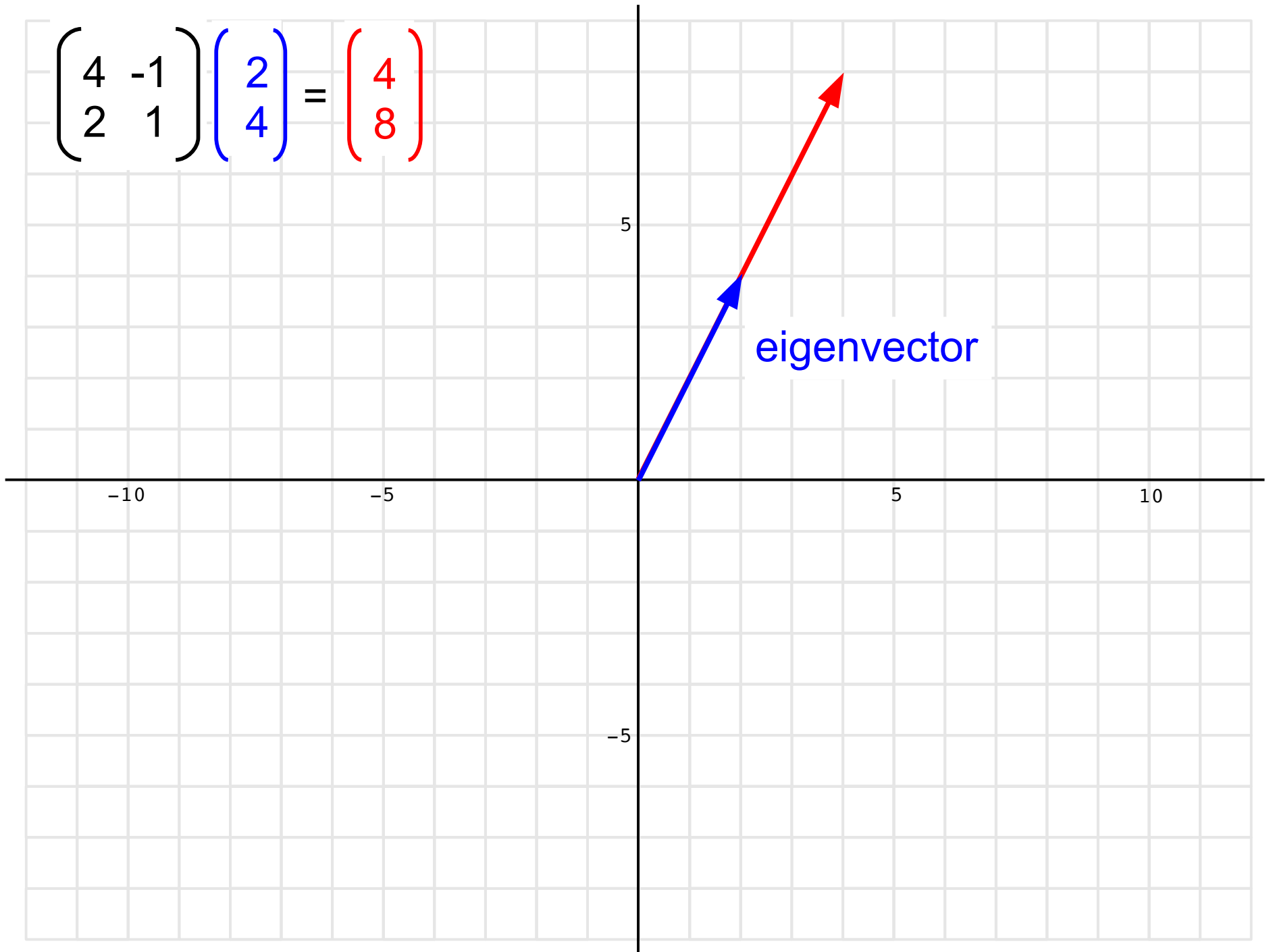
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$



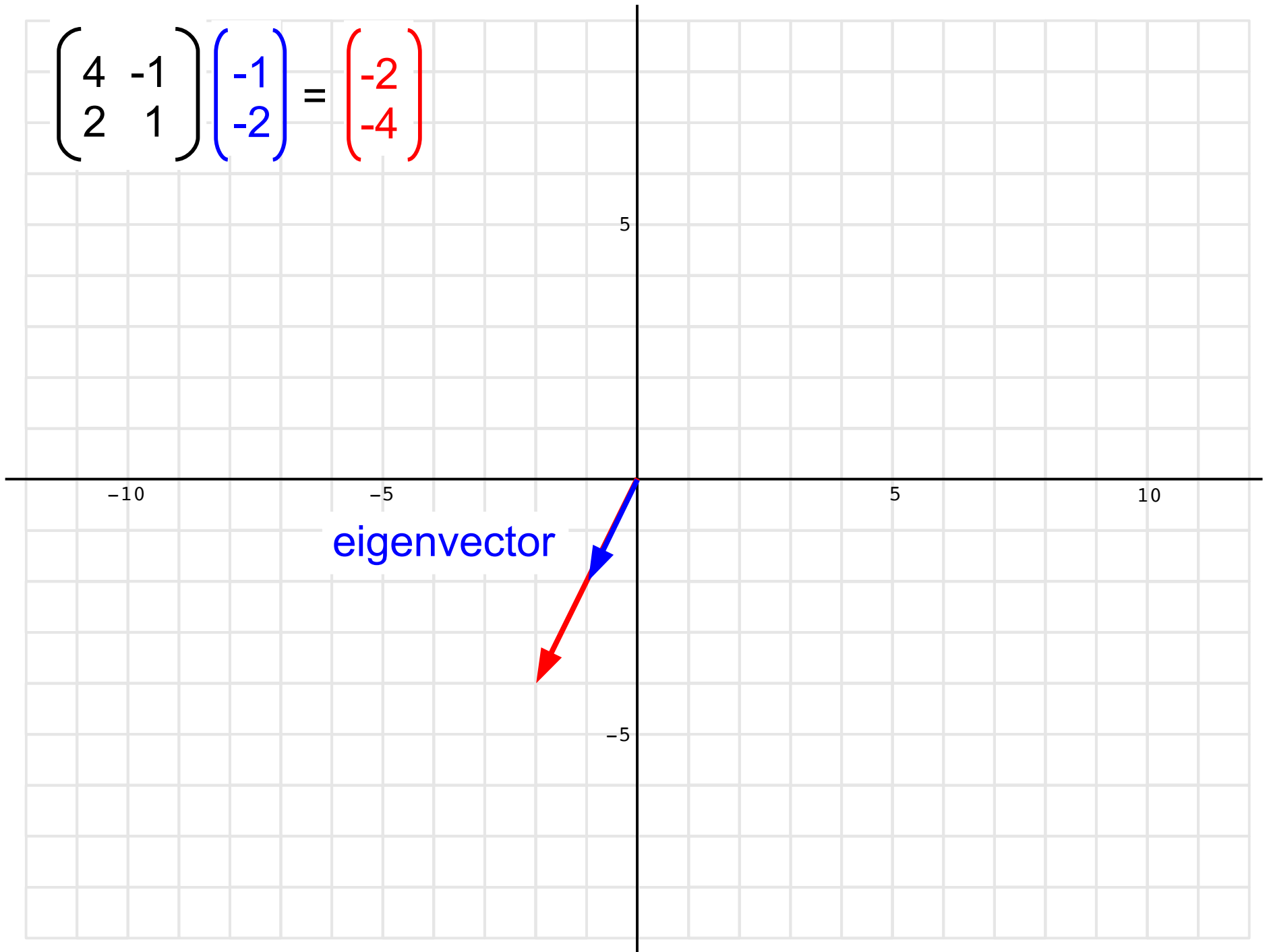
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$



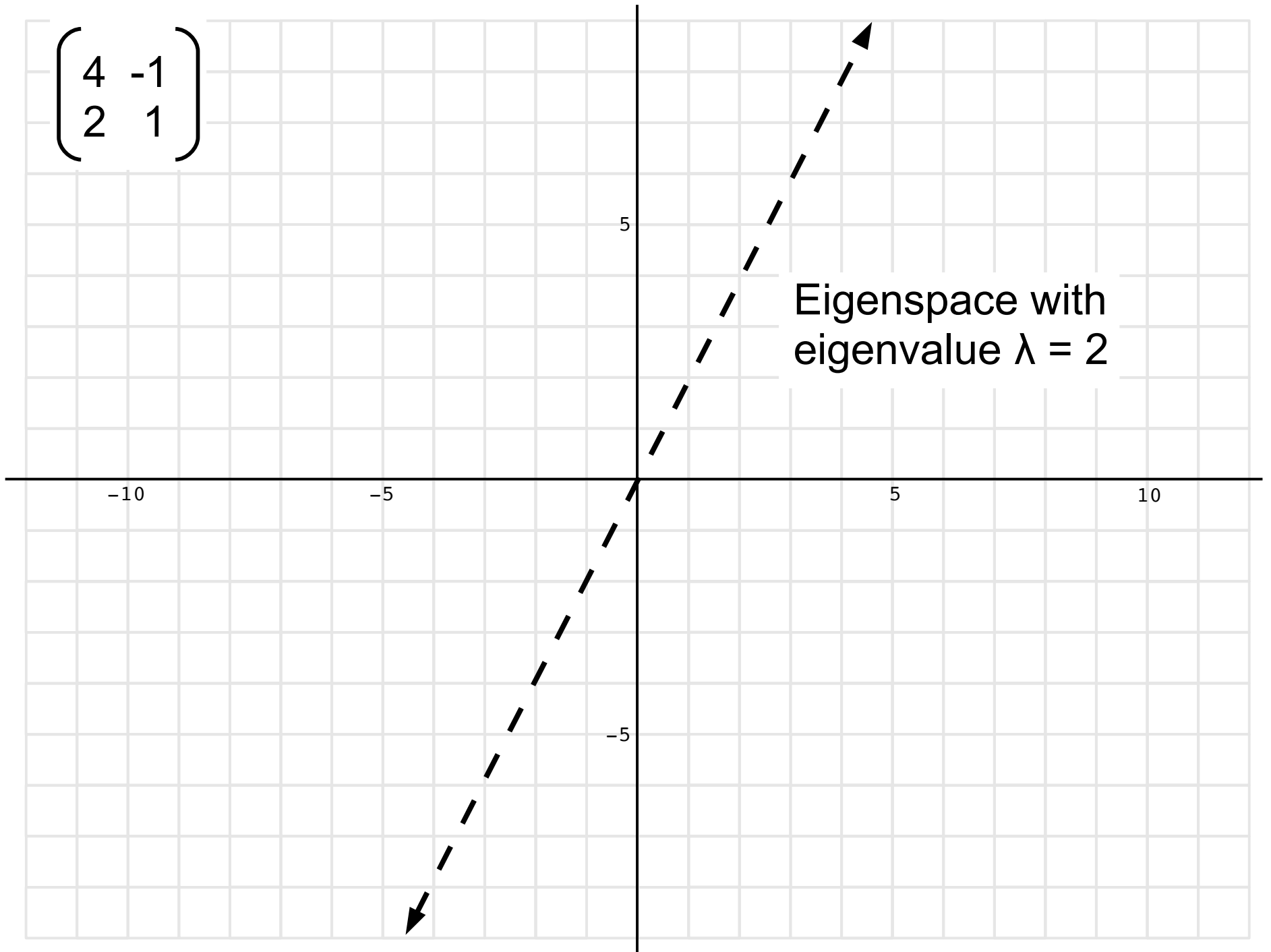
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$



$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$$



$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$



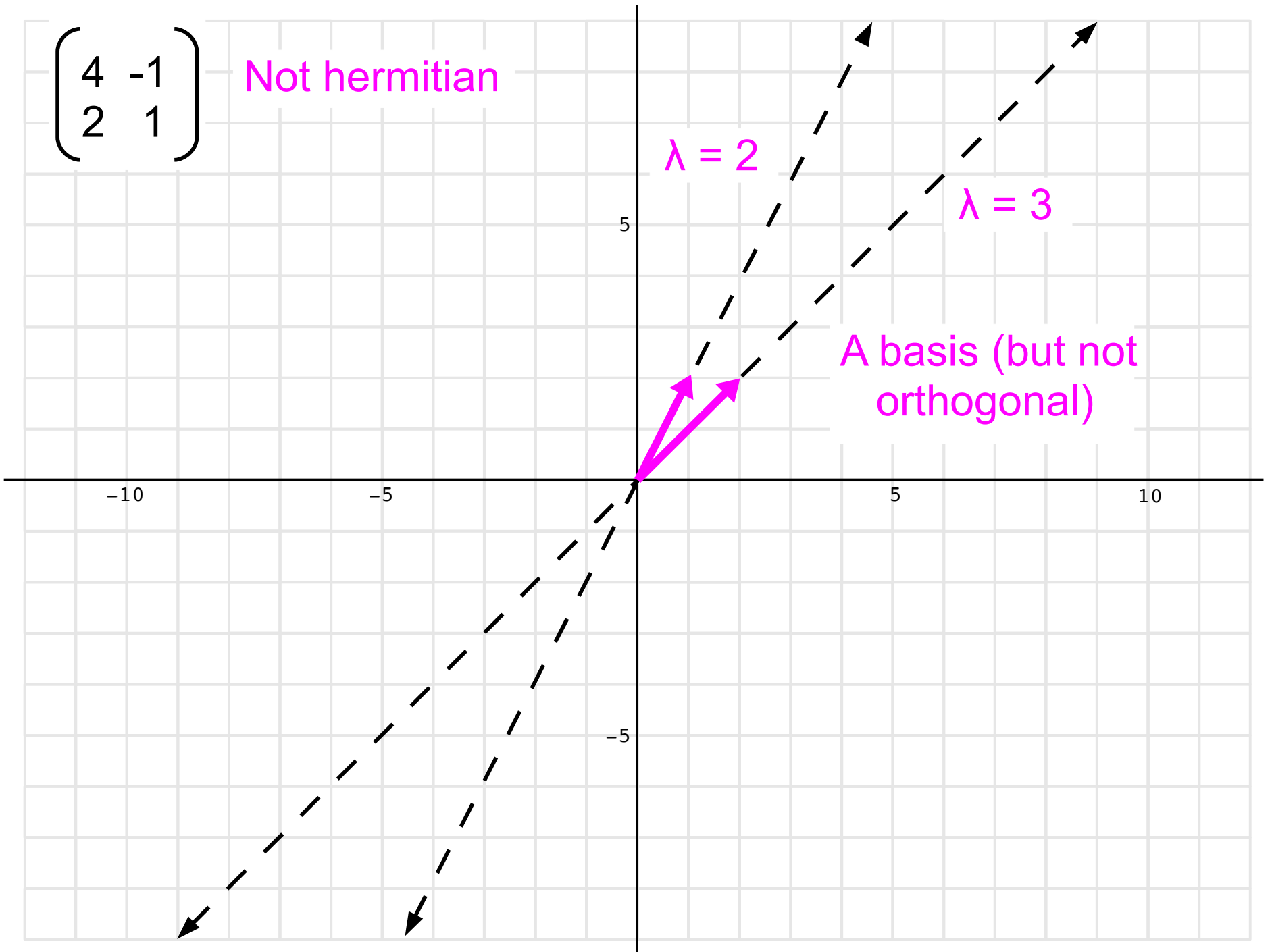
$$\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$

Not hermitian

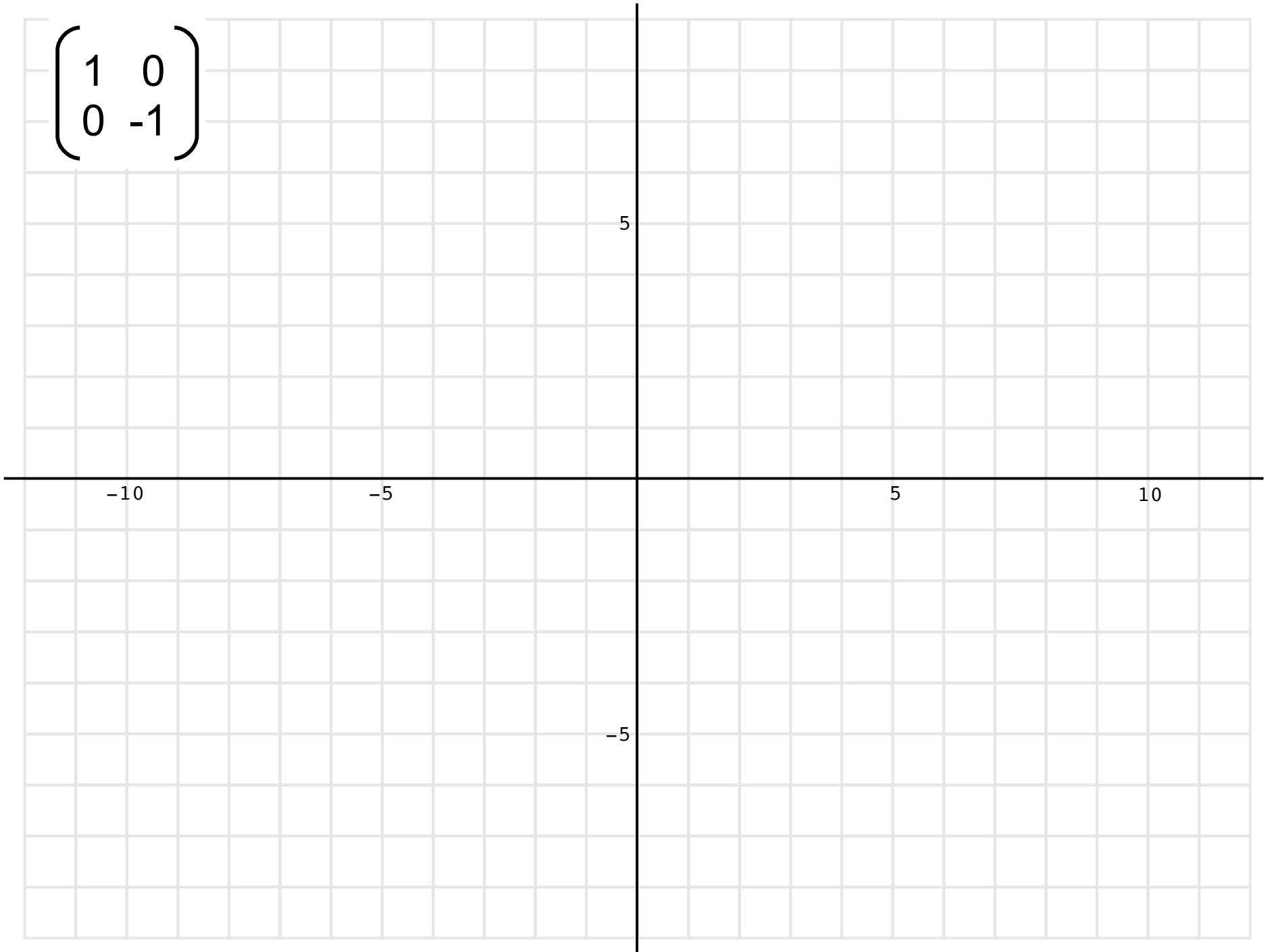
$\lambda = 2$

$\lambda = 3$

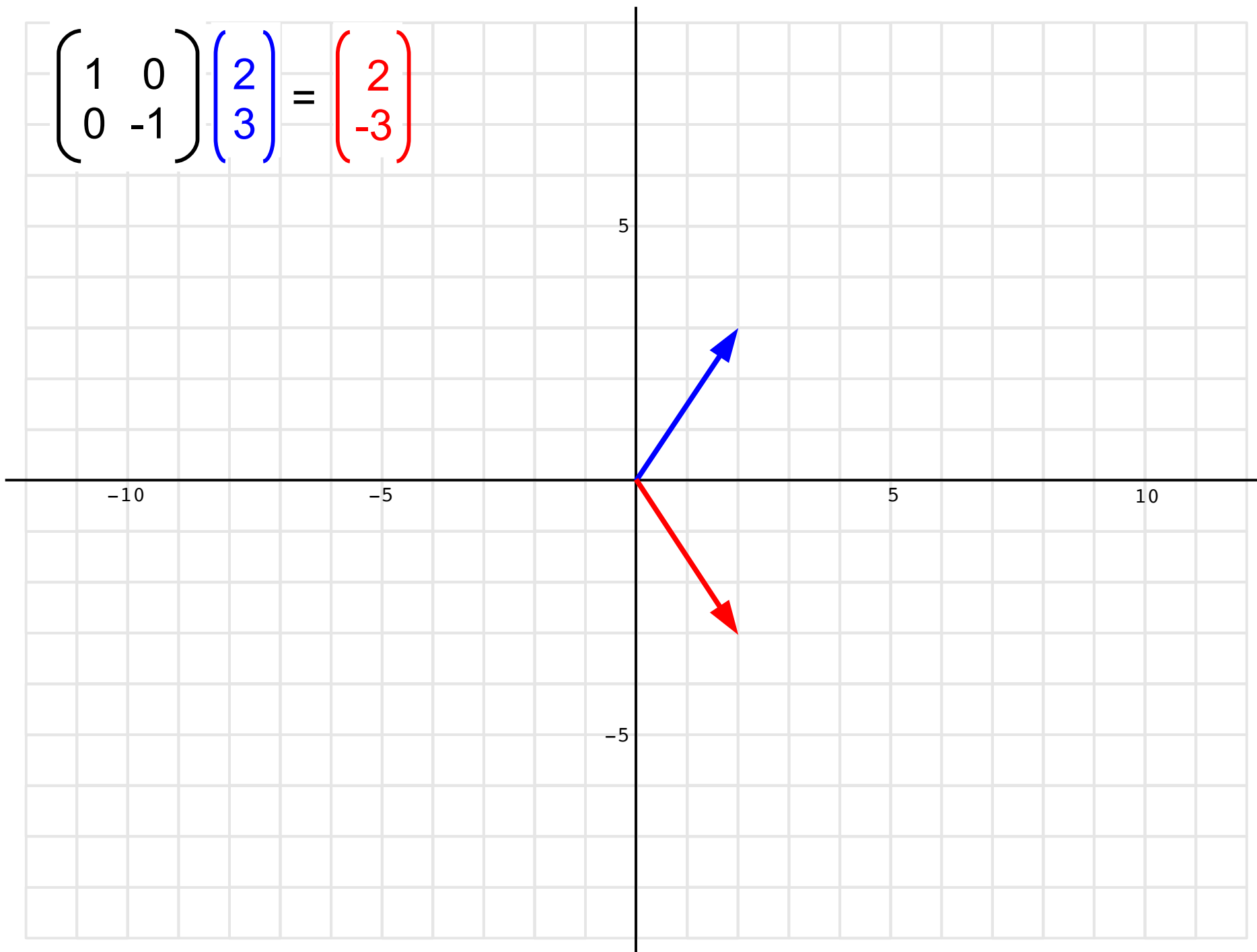
A basis (but not orthogonal)



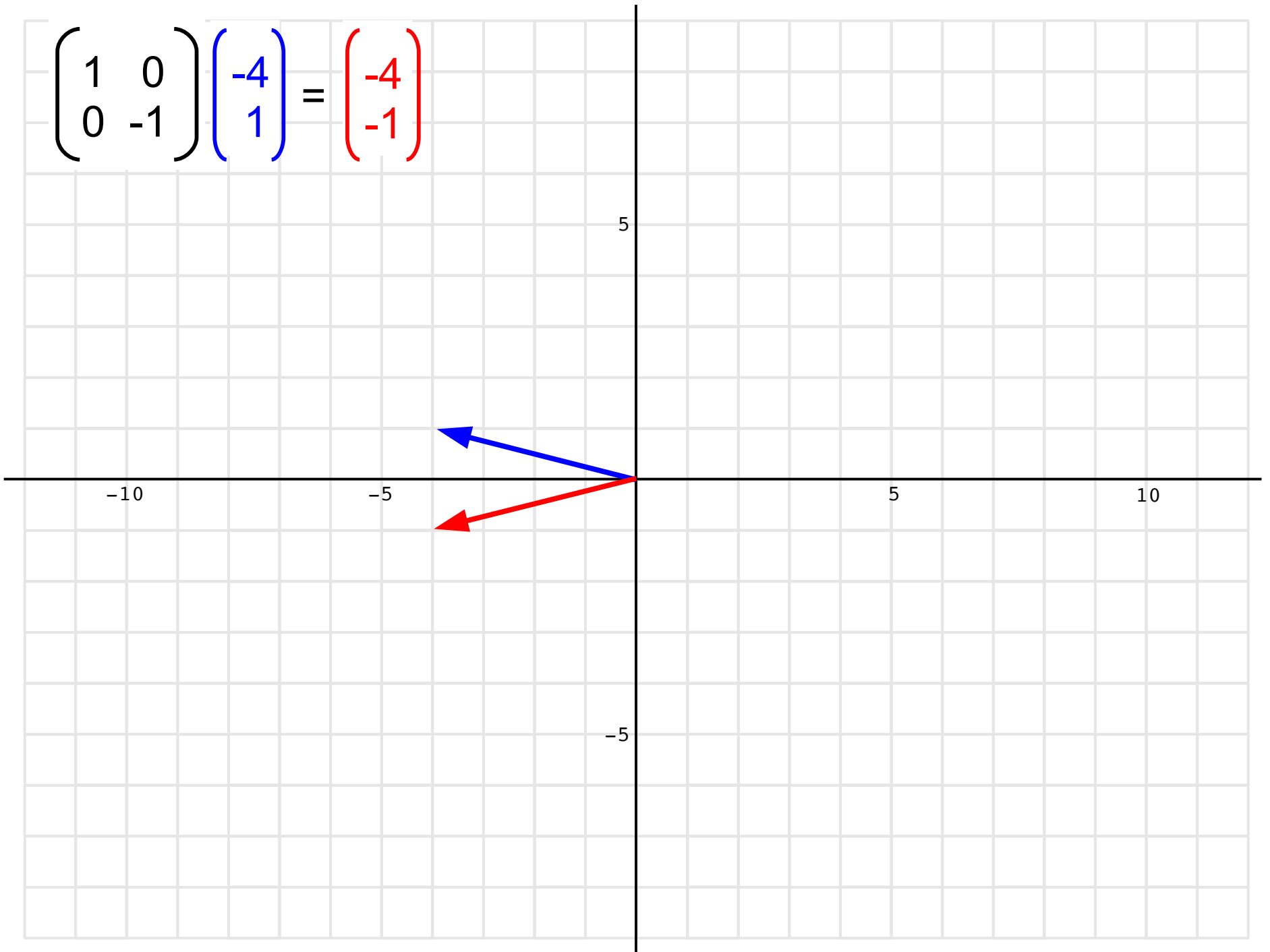
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



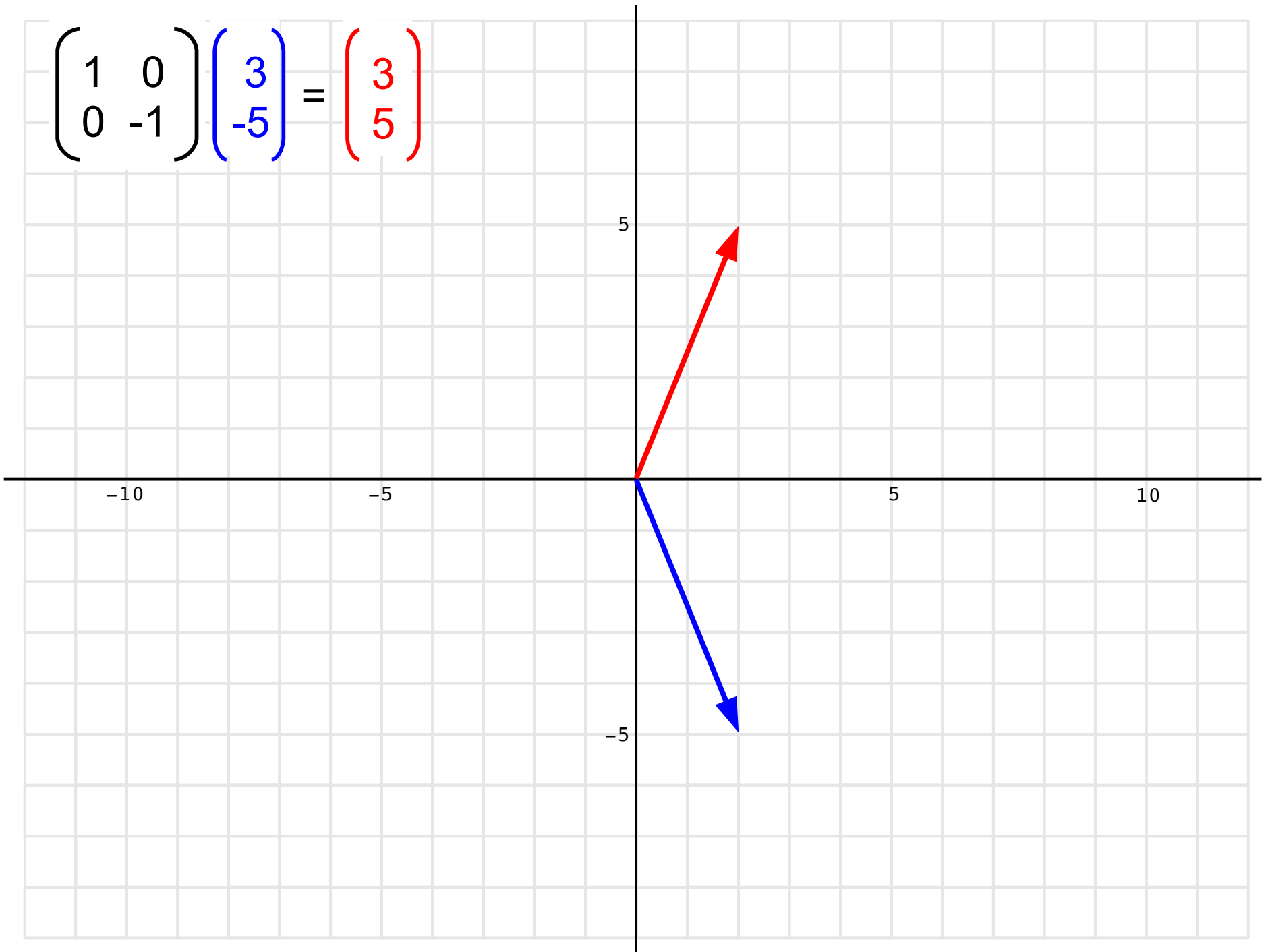
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$$

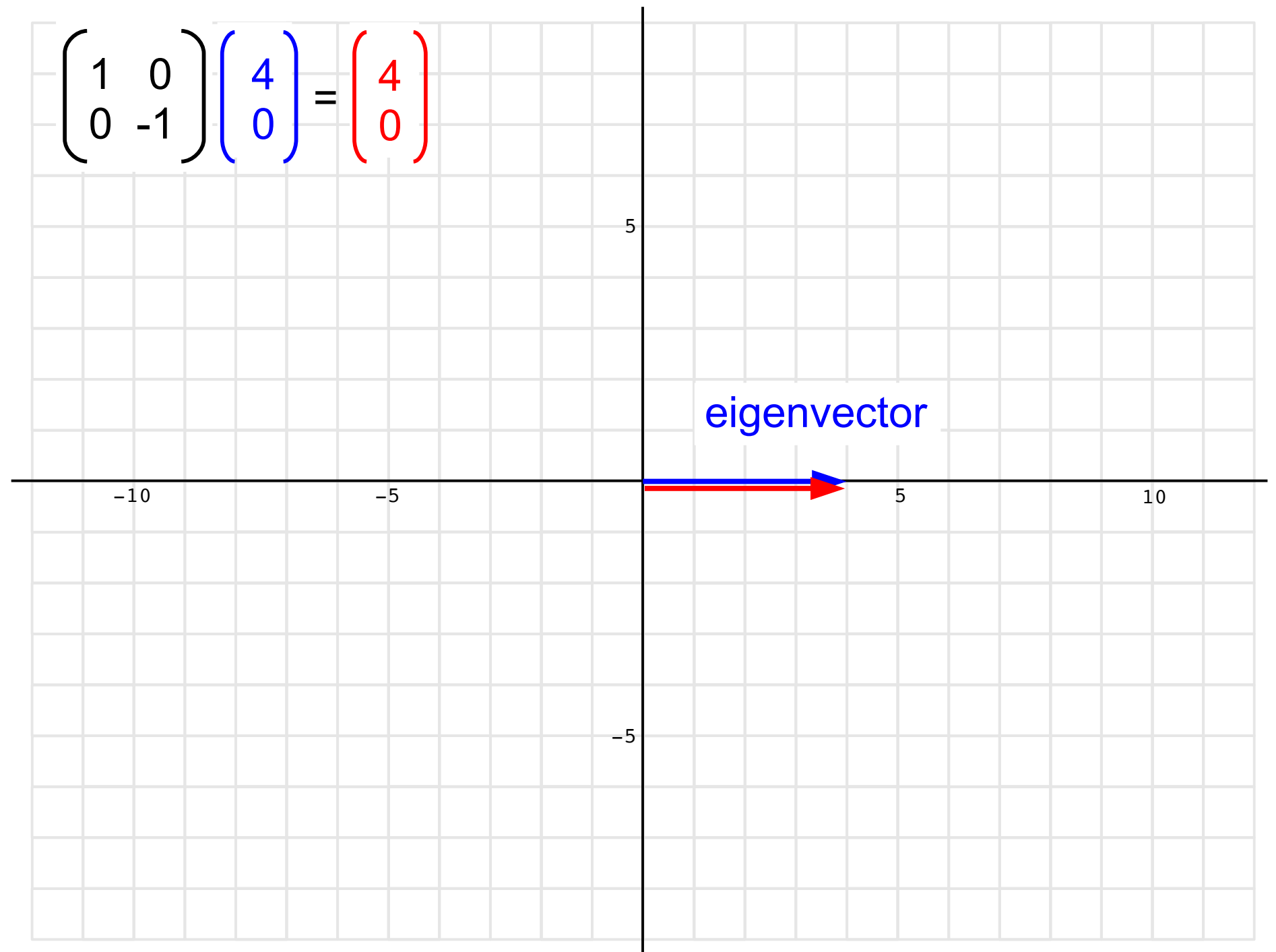


$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

eigenvector



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

eigenvector



-10

-5

5

10

-5

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$$

eigenvector



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

5

Eigenspace with
eigenvalue $\lambda = 1$

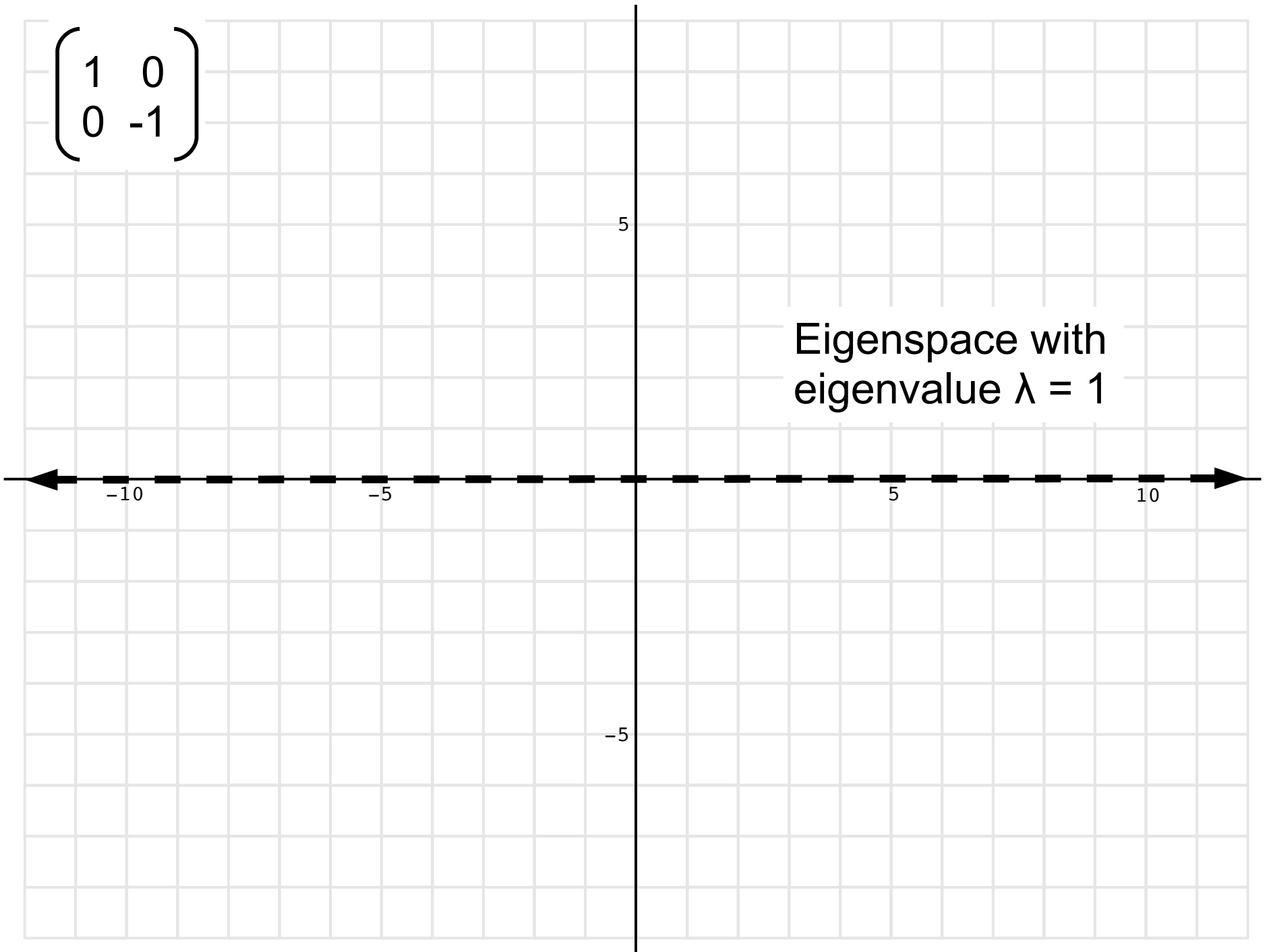
-10

-5

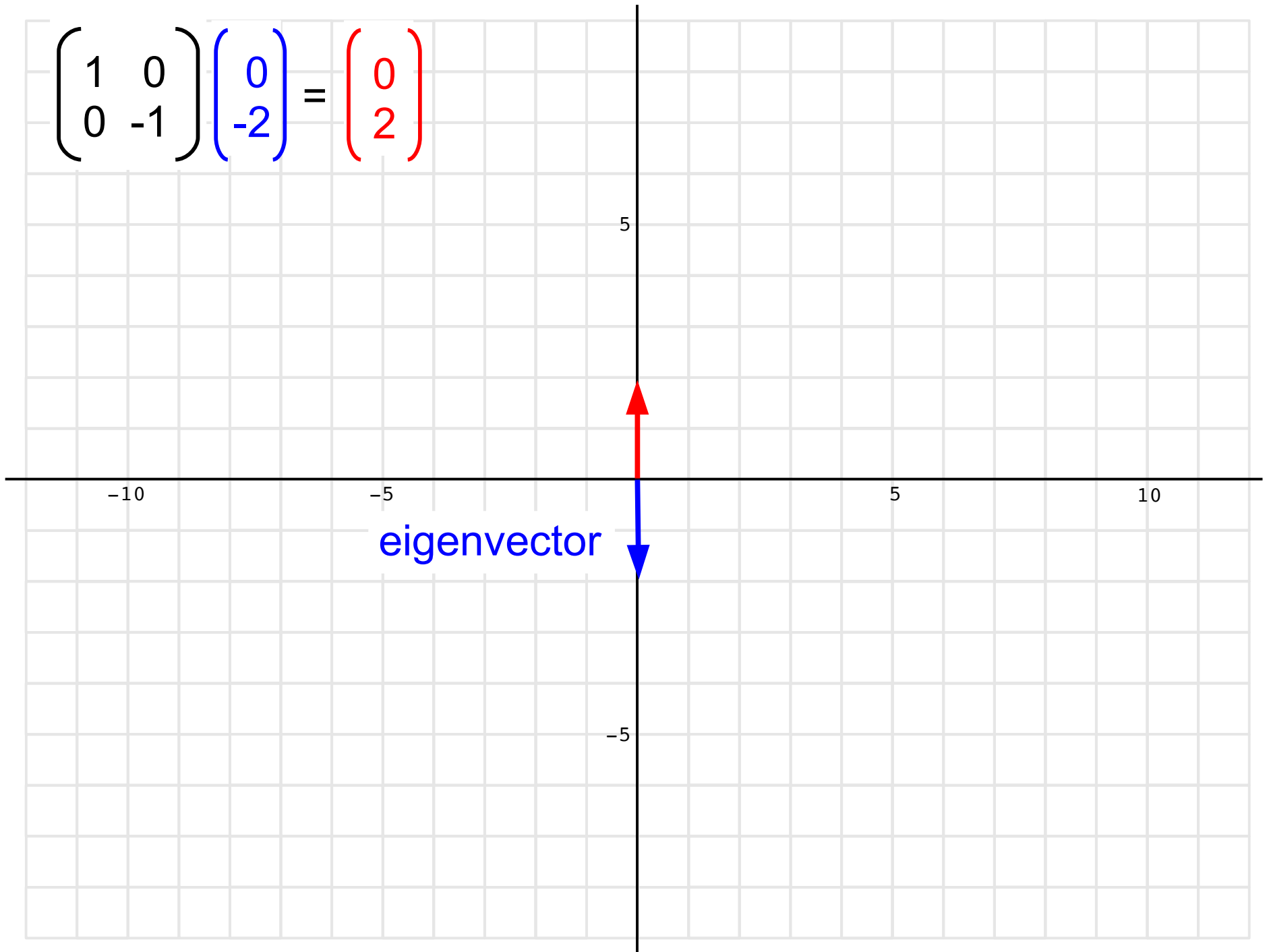
5

10

-5



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$$

eigenvector

5

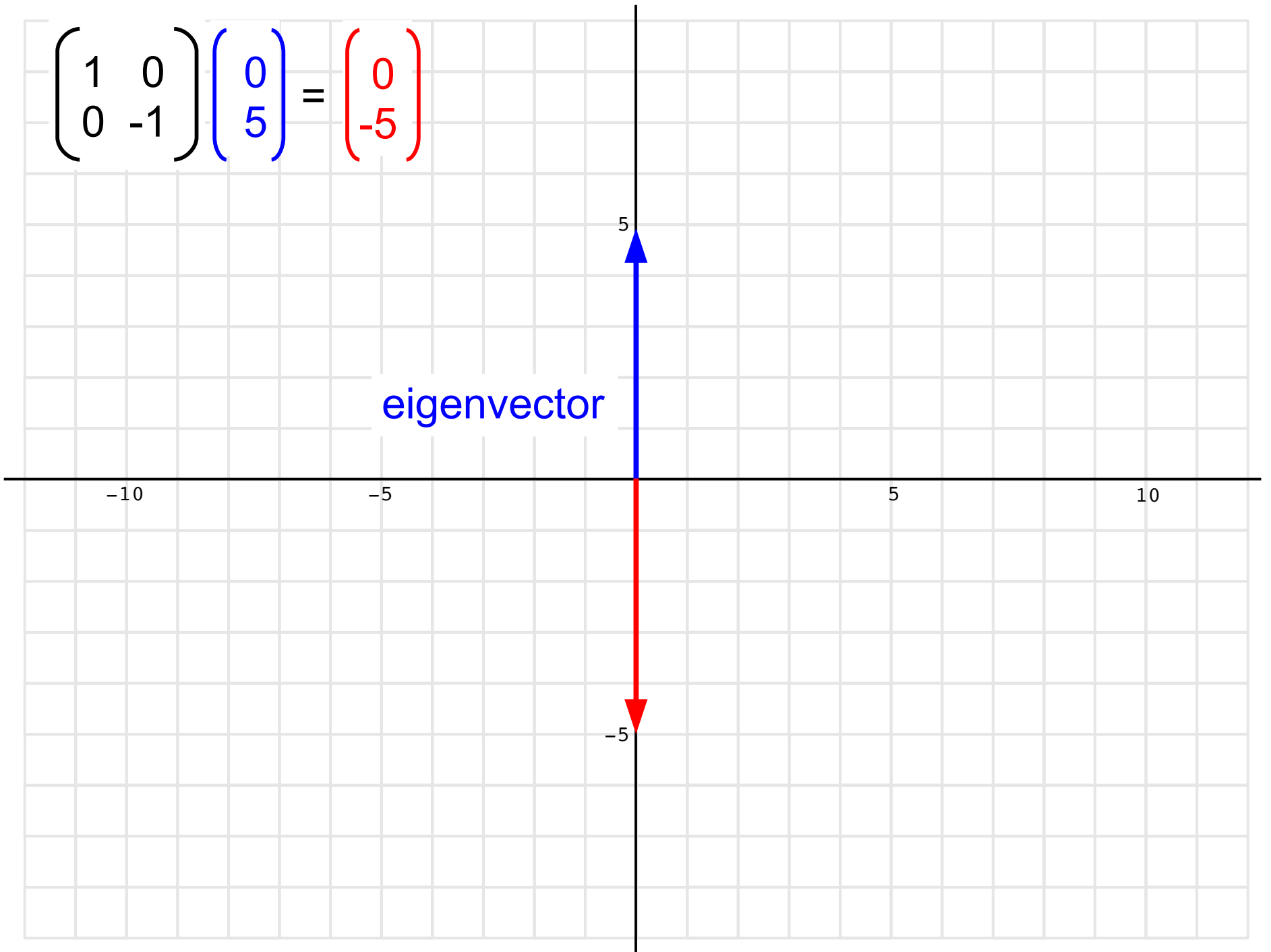
-5

-10

-5

5

10



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

eigenvector

5

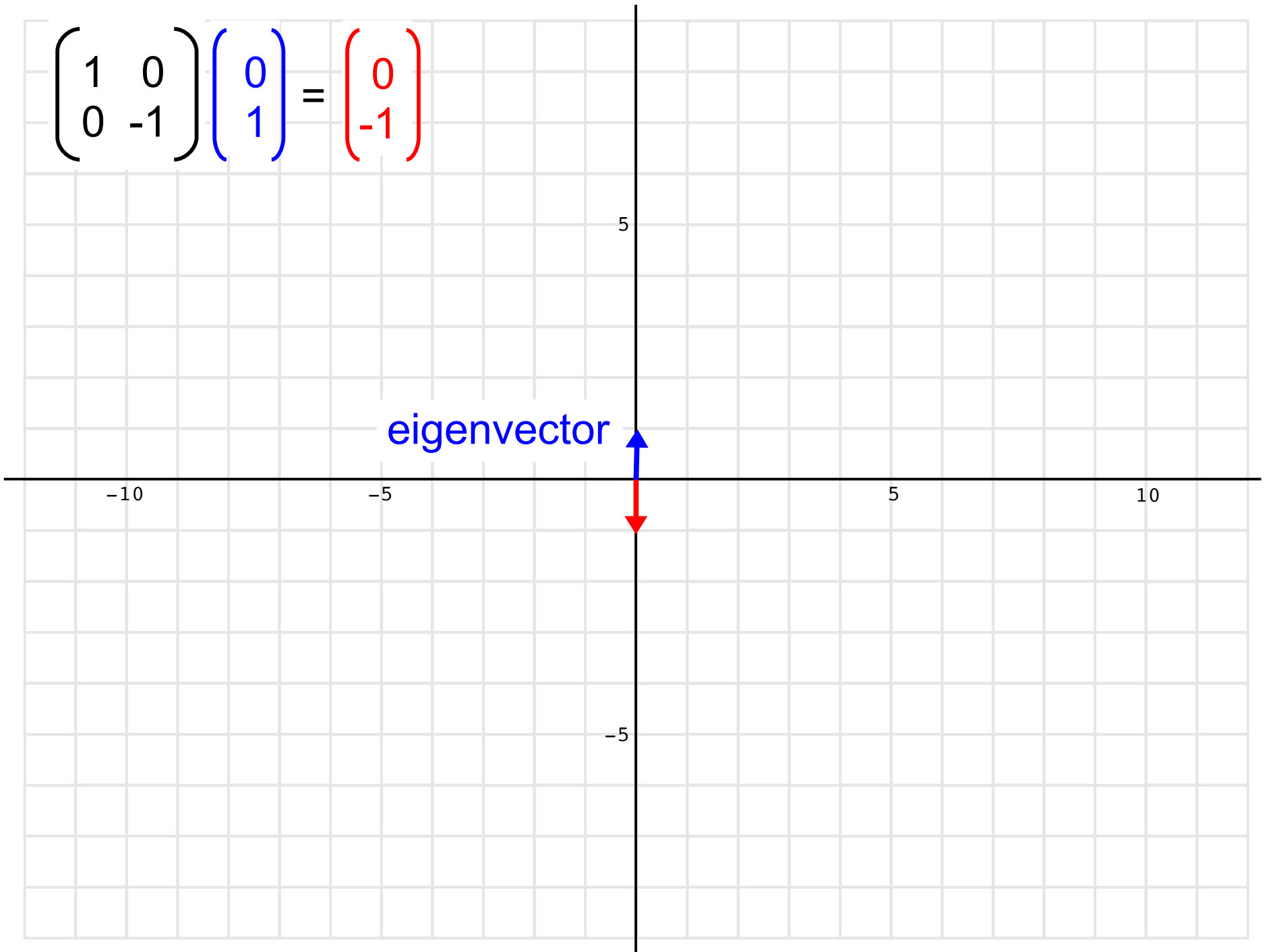
-10

-5

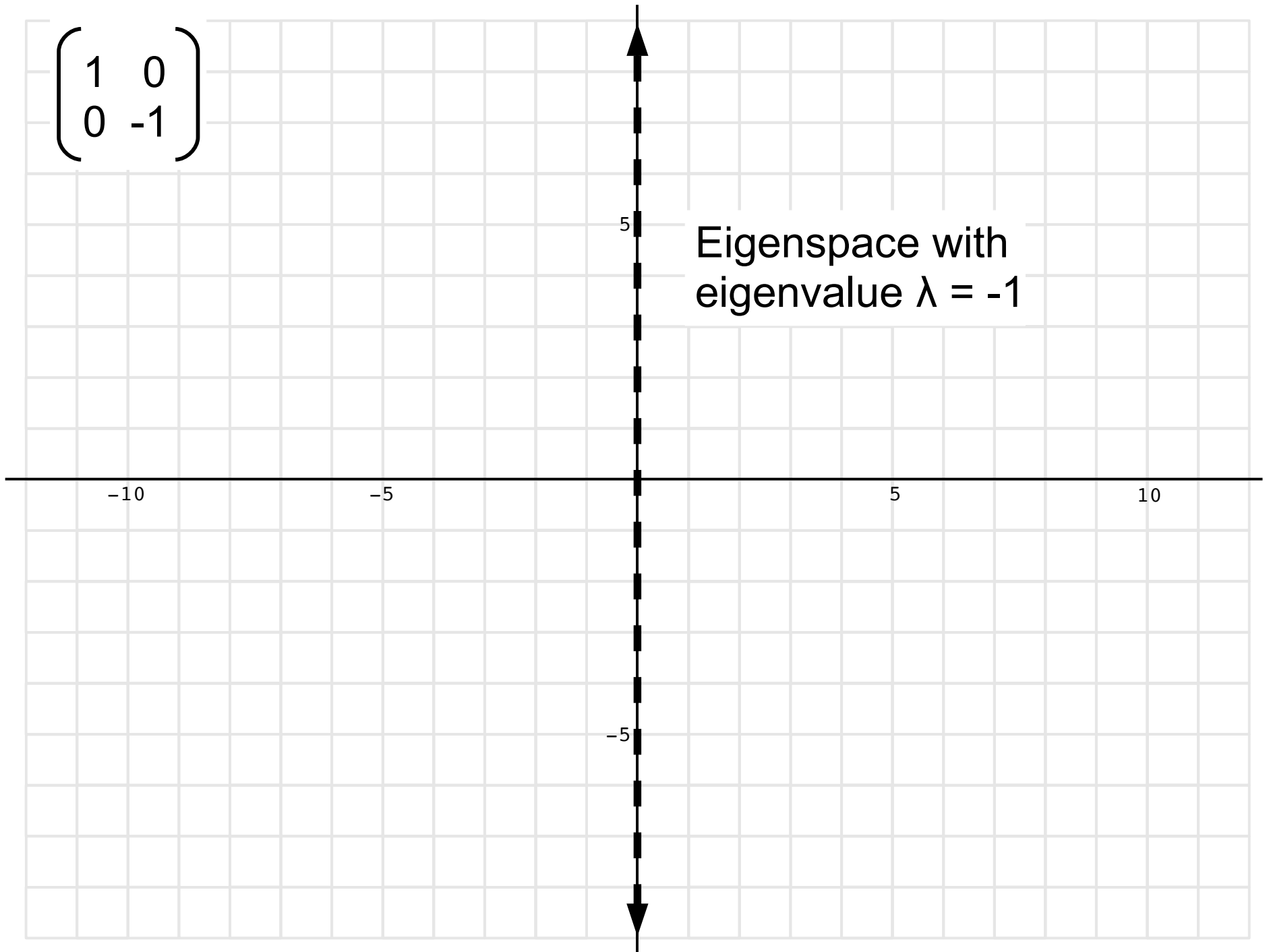
5

10

-5



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Hermitian

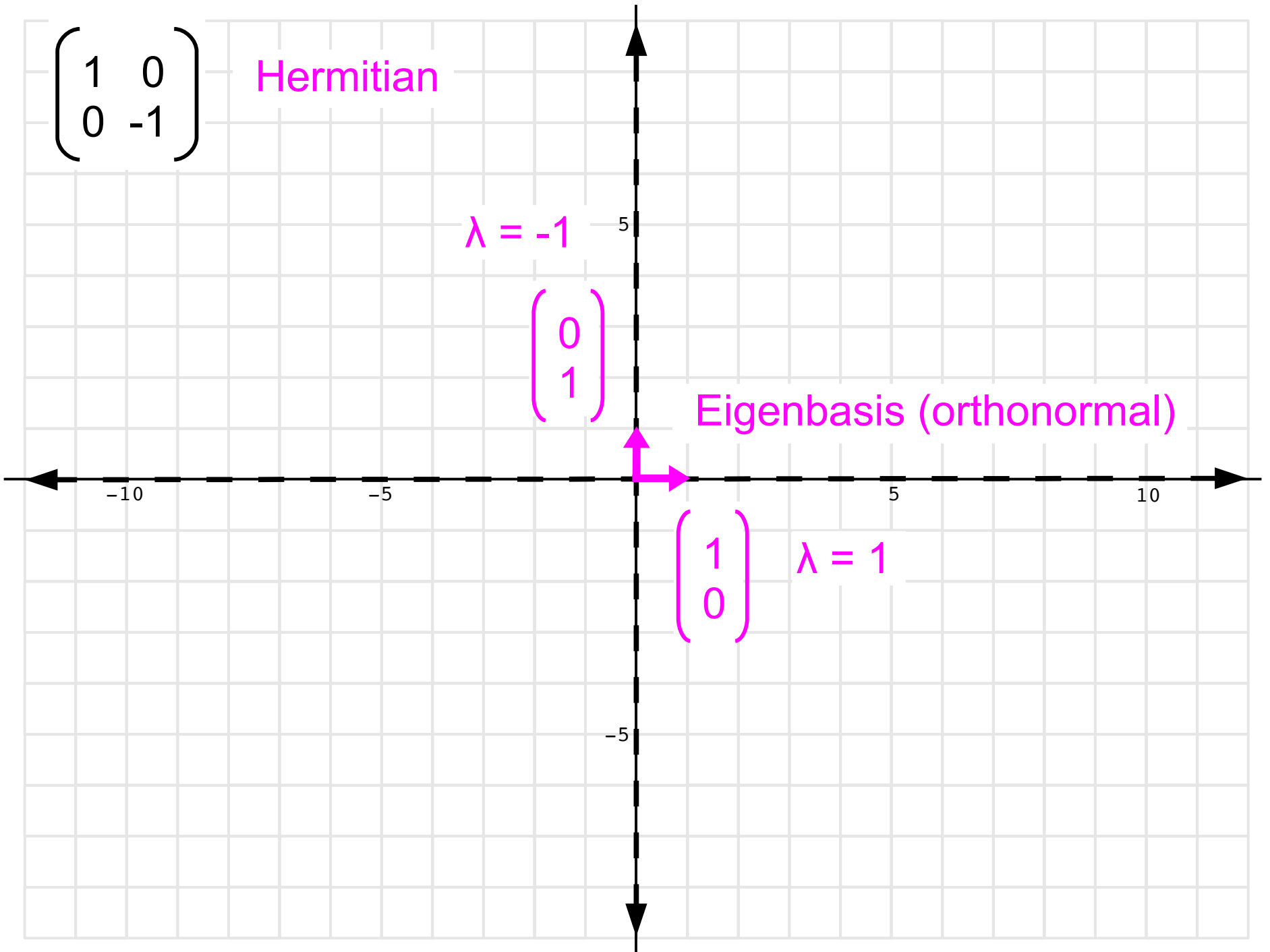
$$\lambda = -1$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

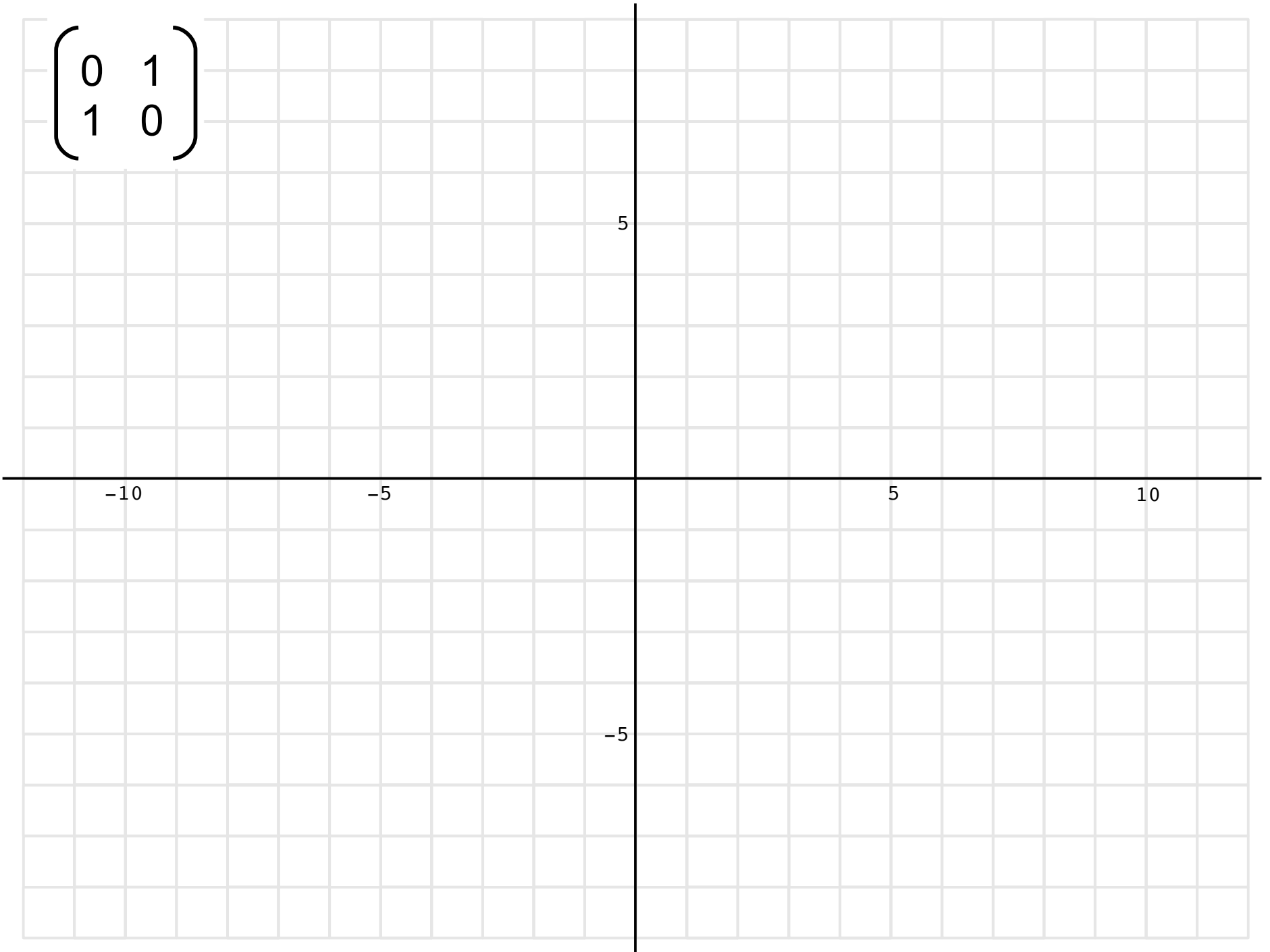
Eigenbasis (orthonormal)

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

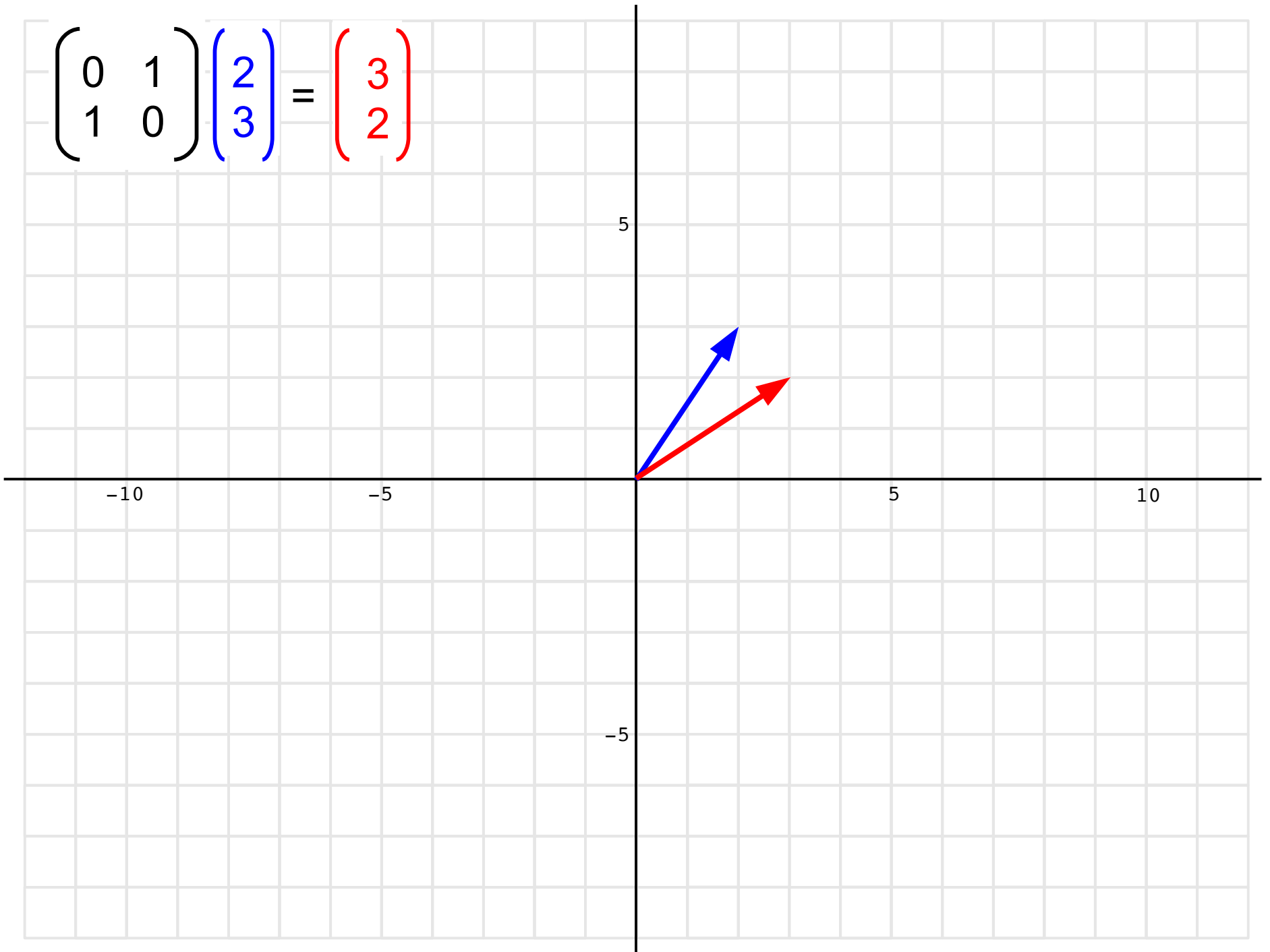
$$\lambda = 1$$



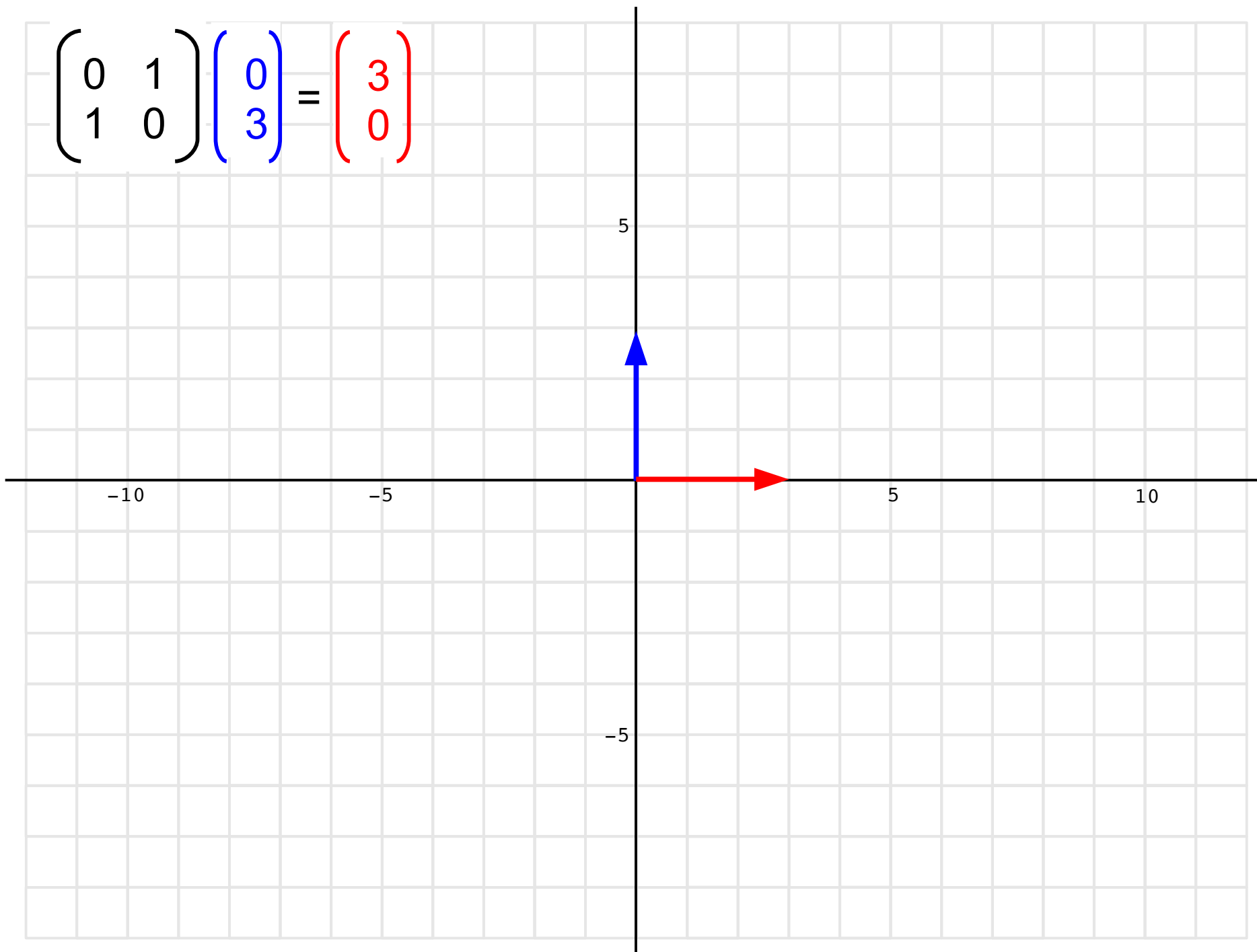
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



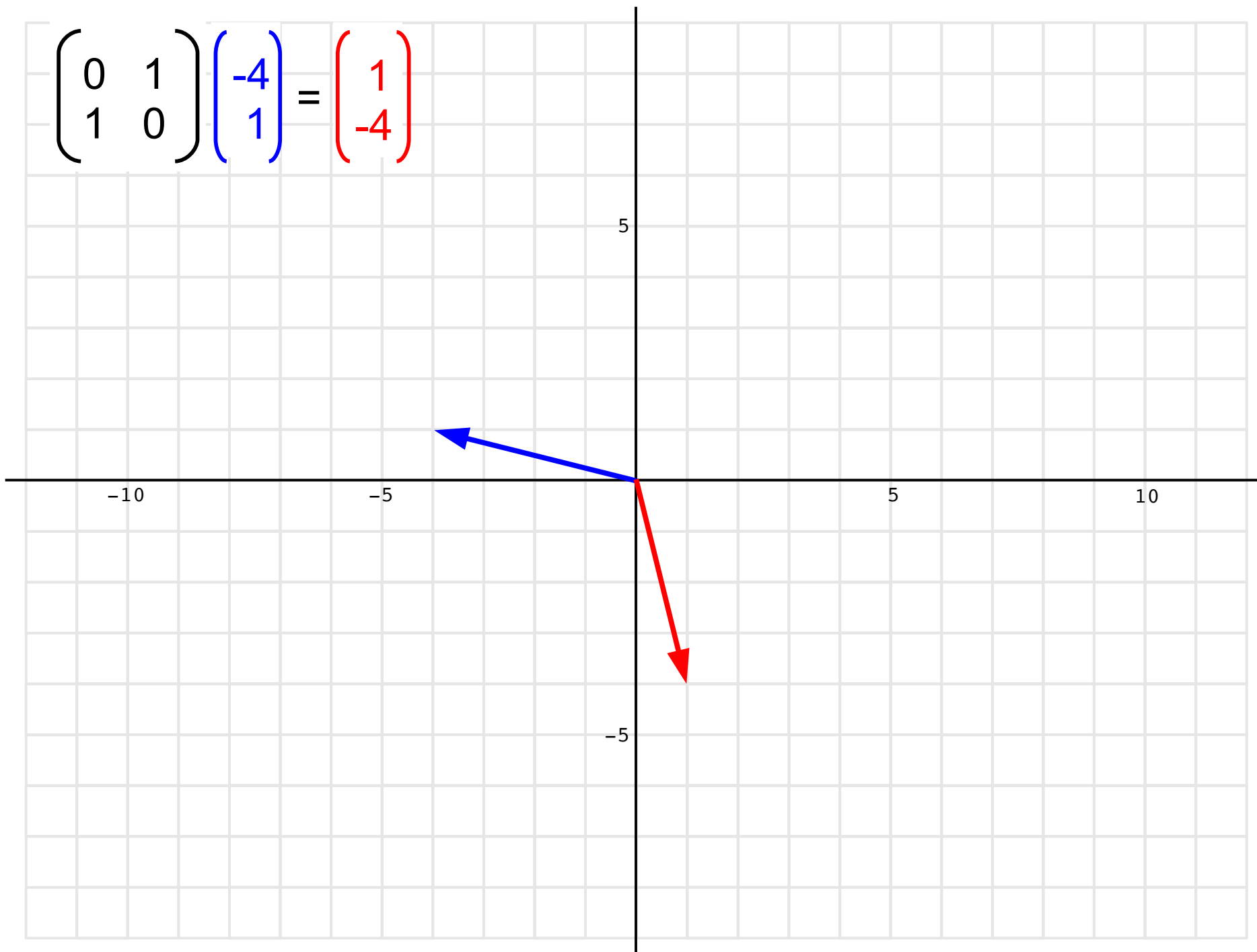
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



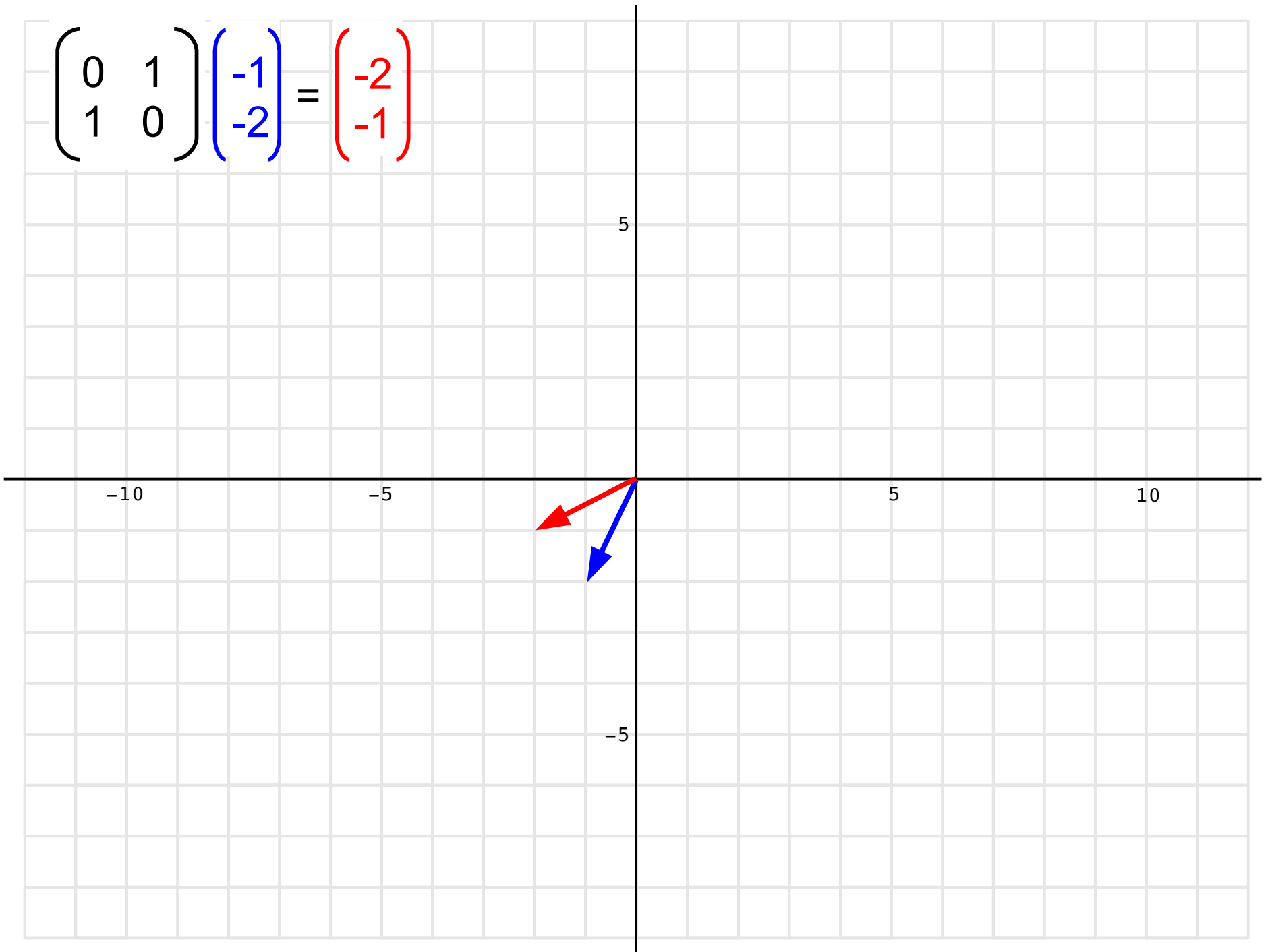
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$



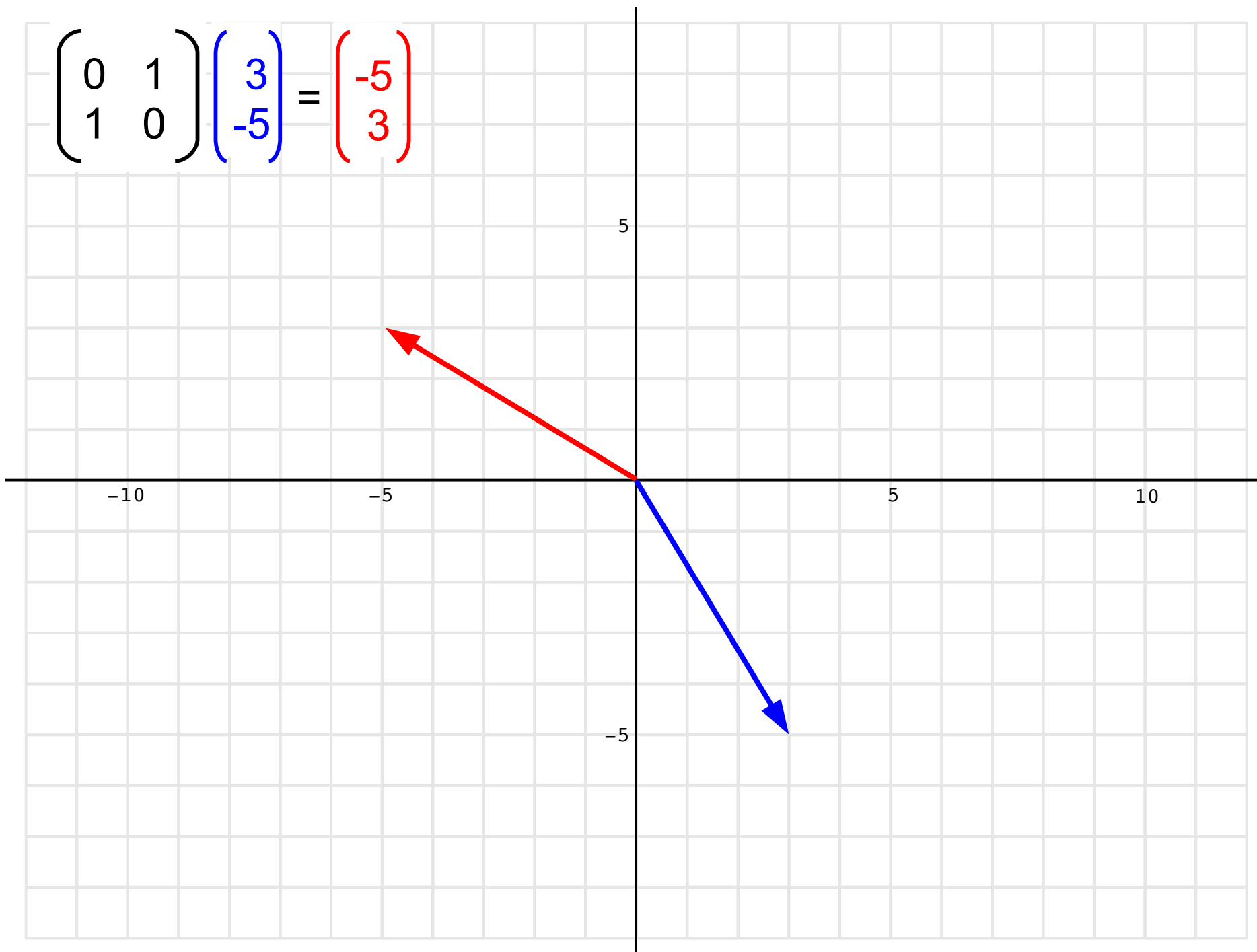
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

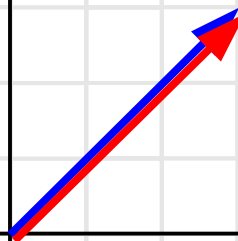


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

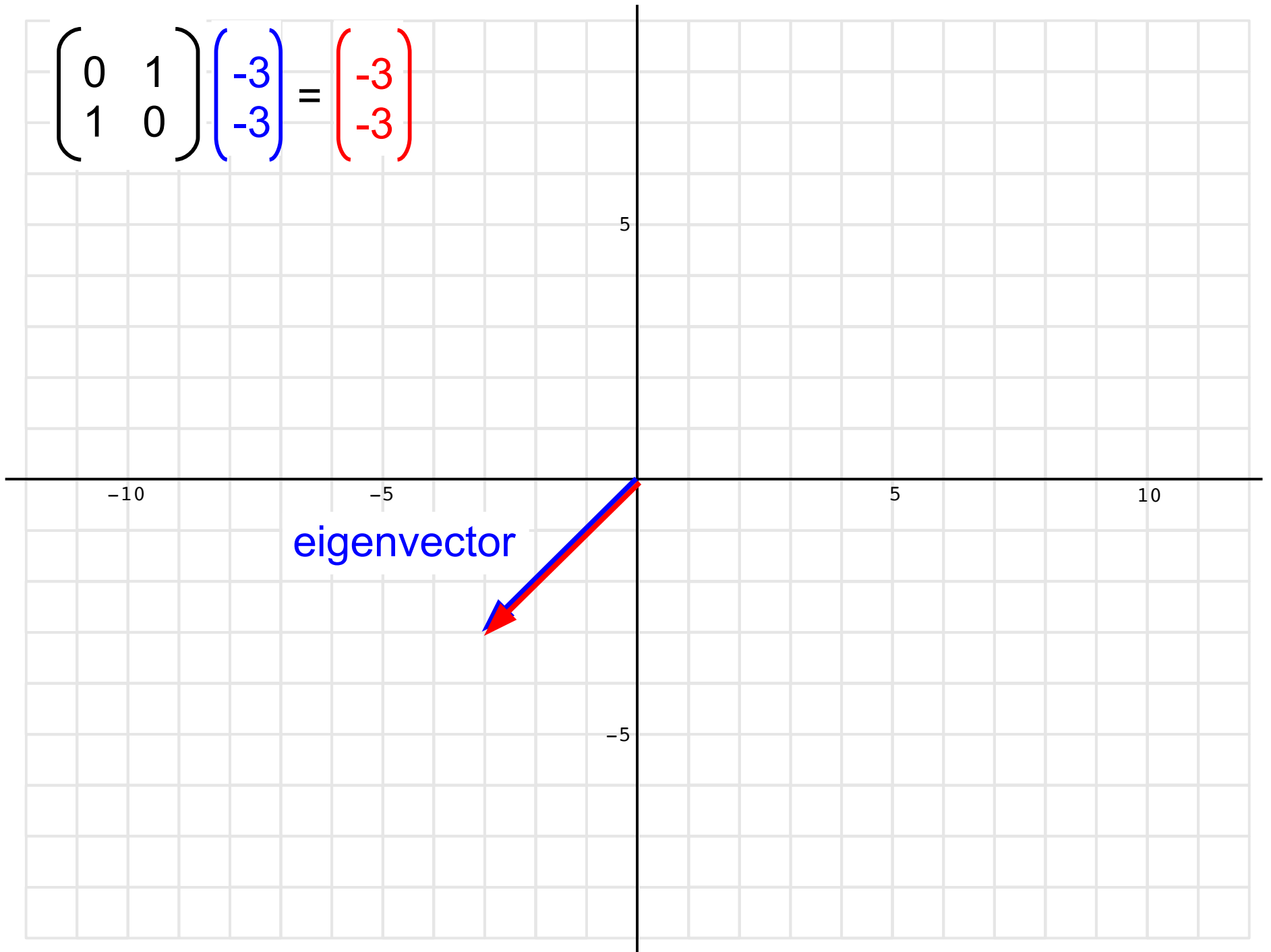


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

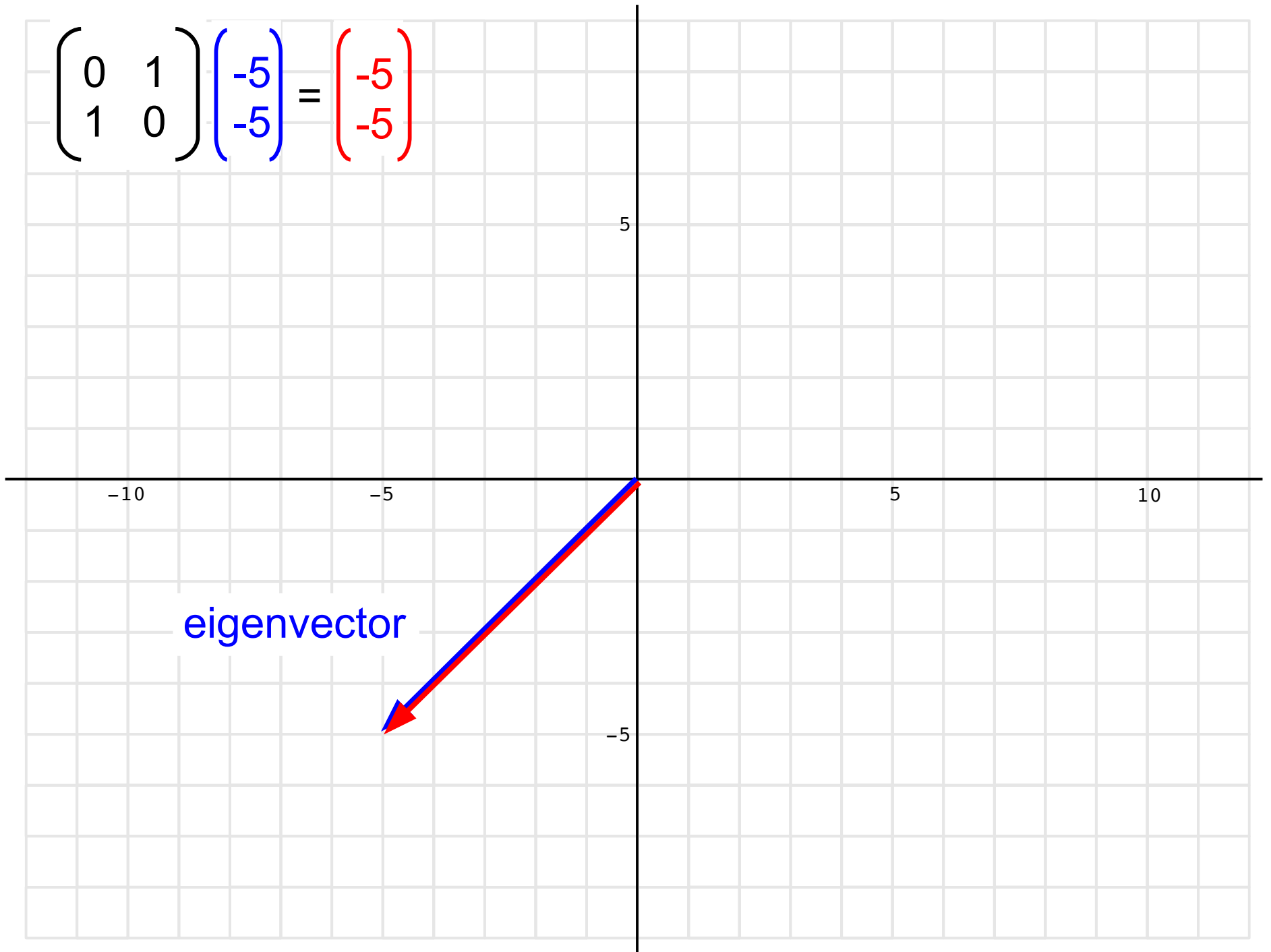
eigenvector



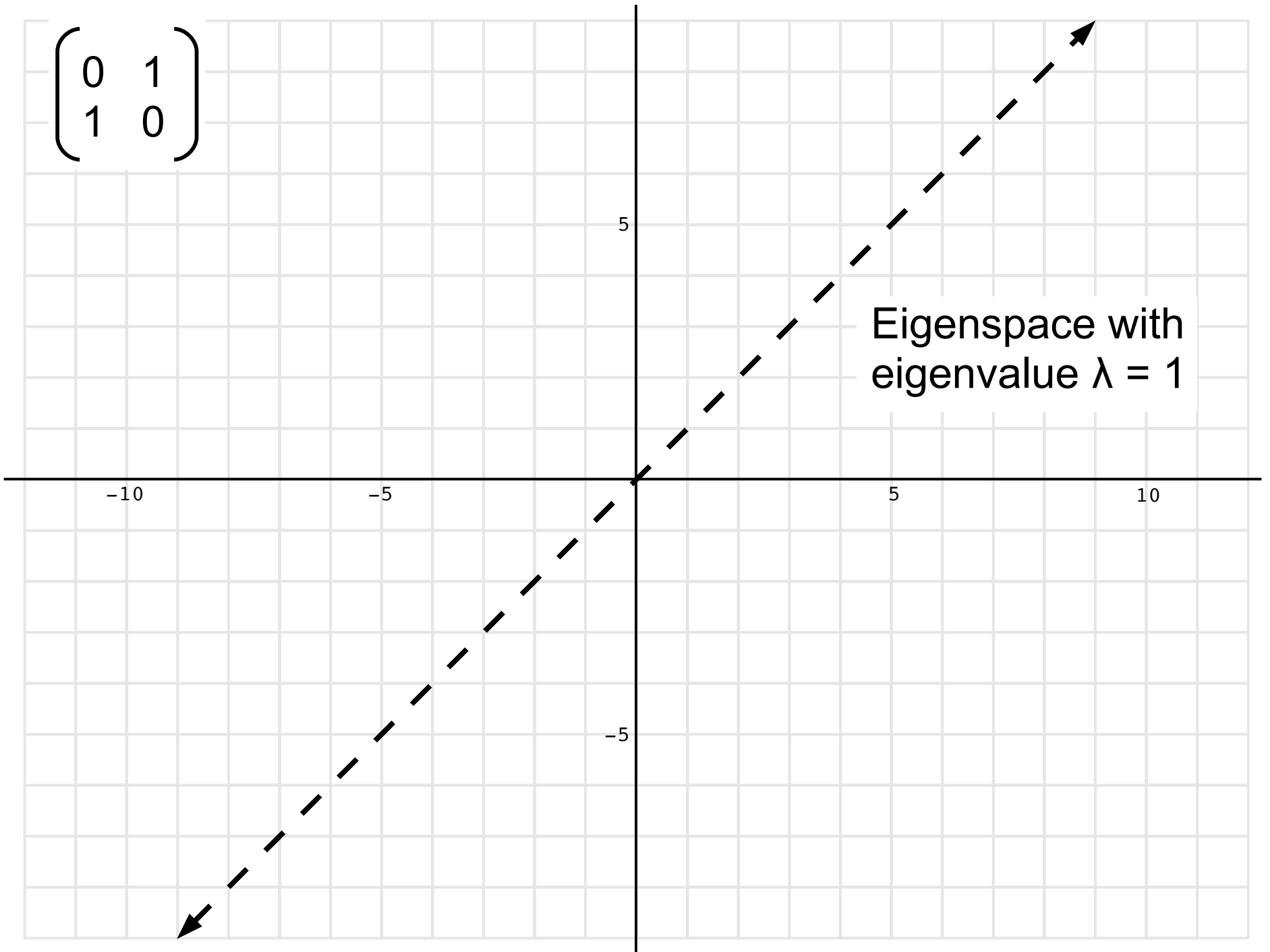
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$$



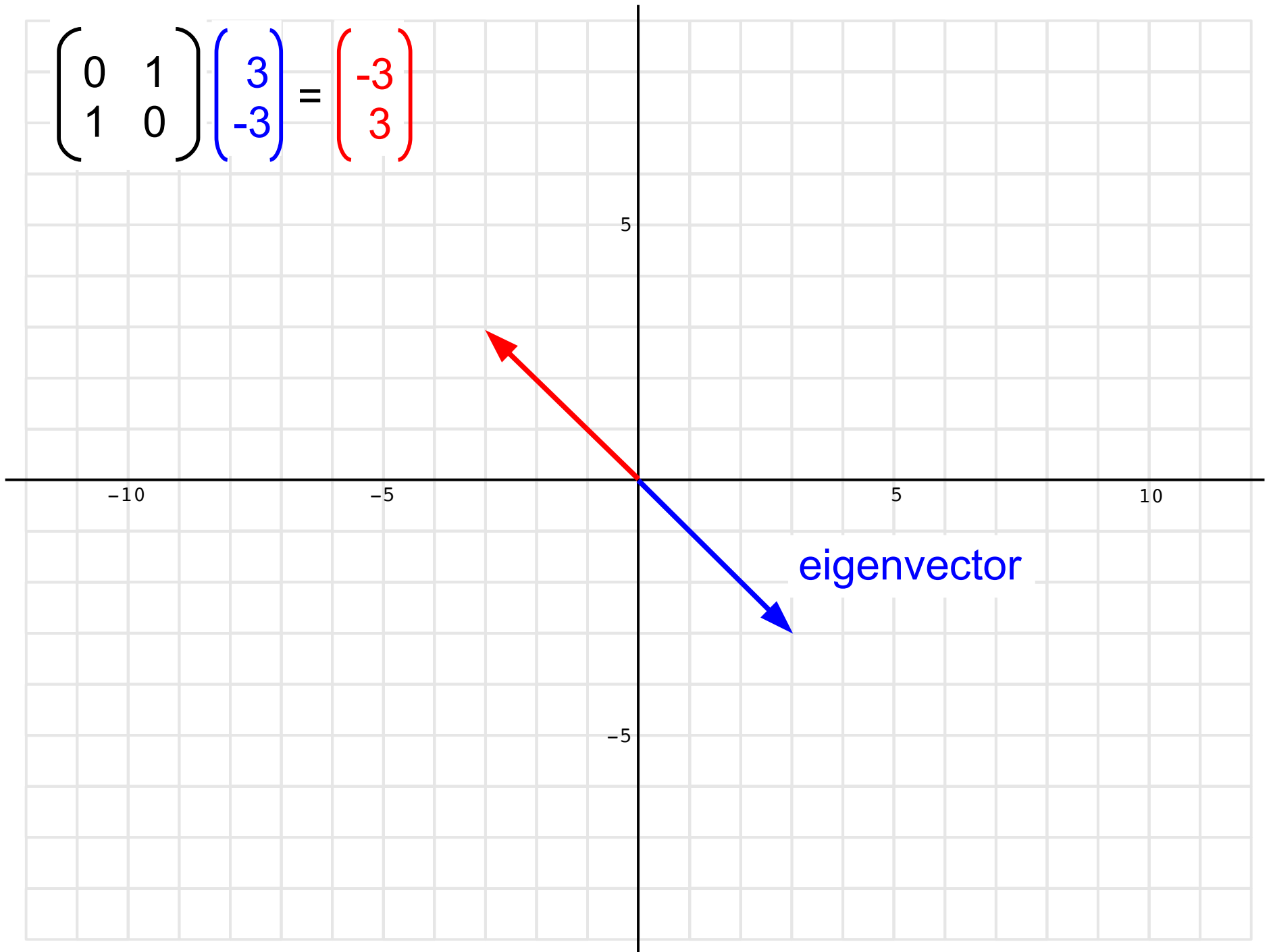
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$$



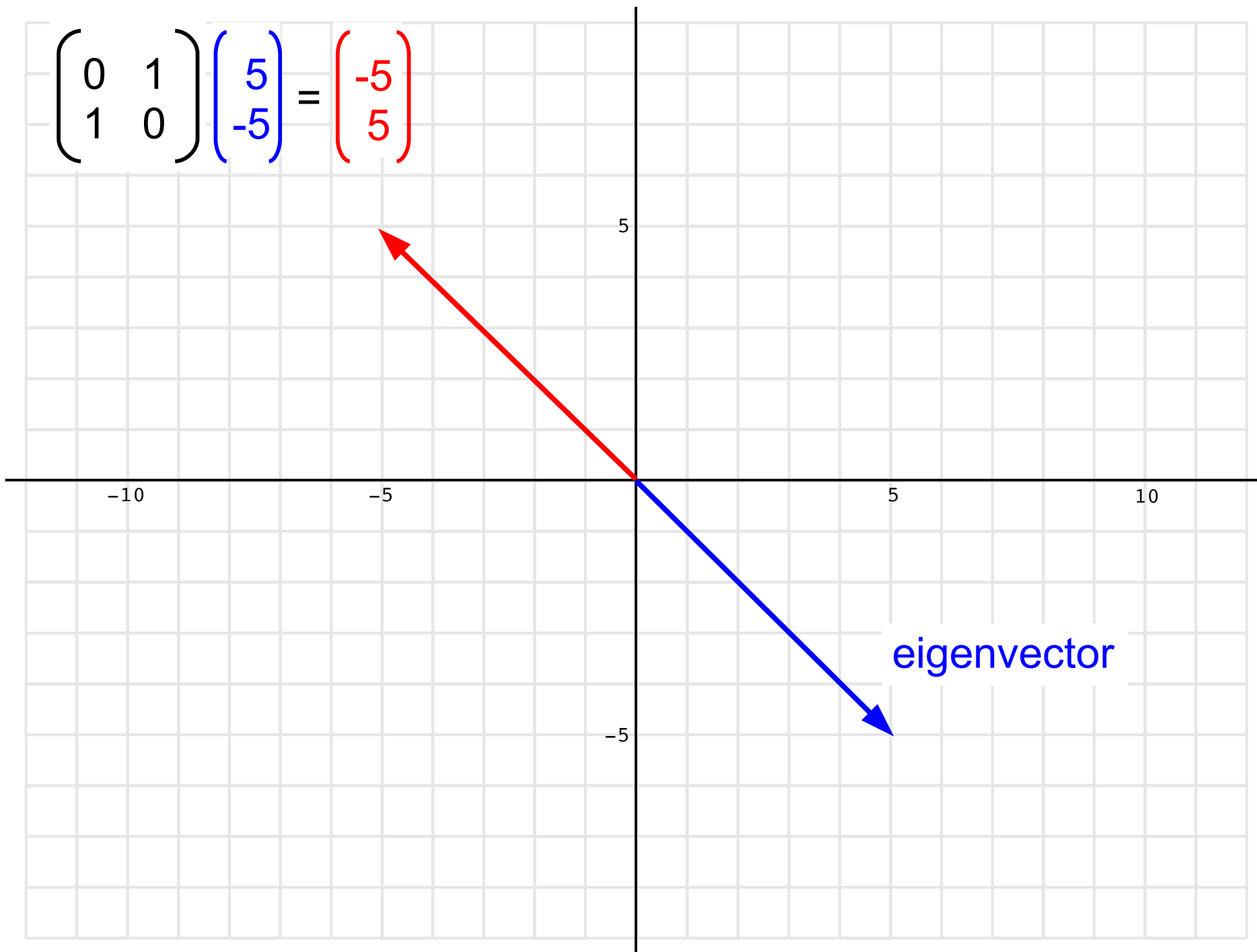
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

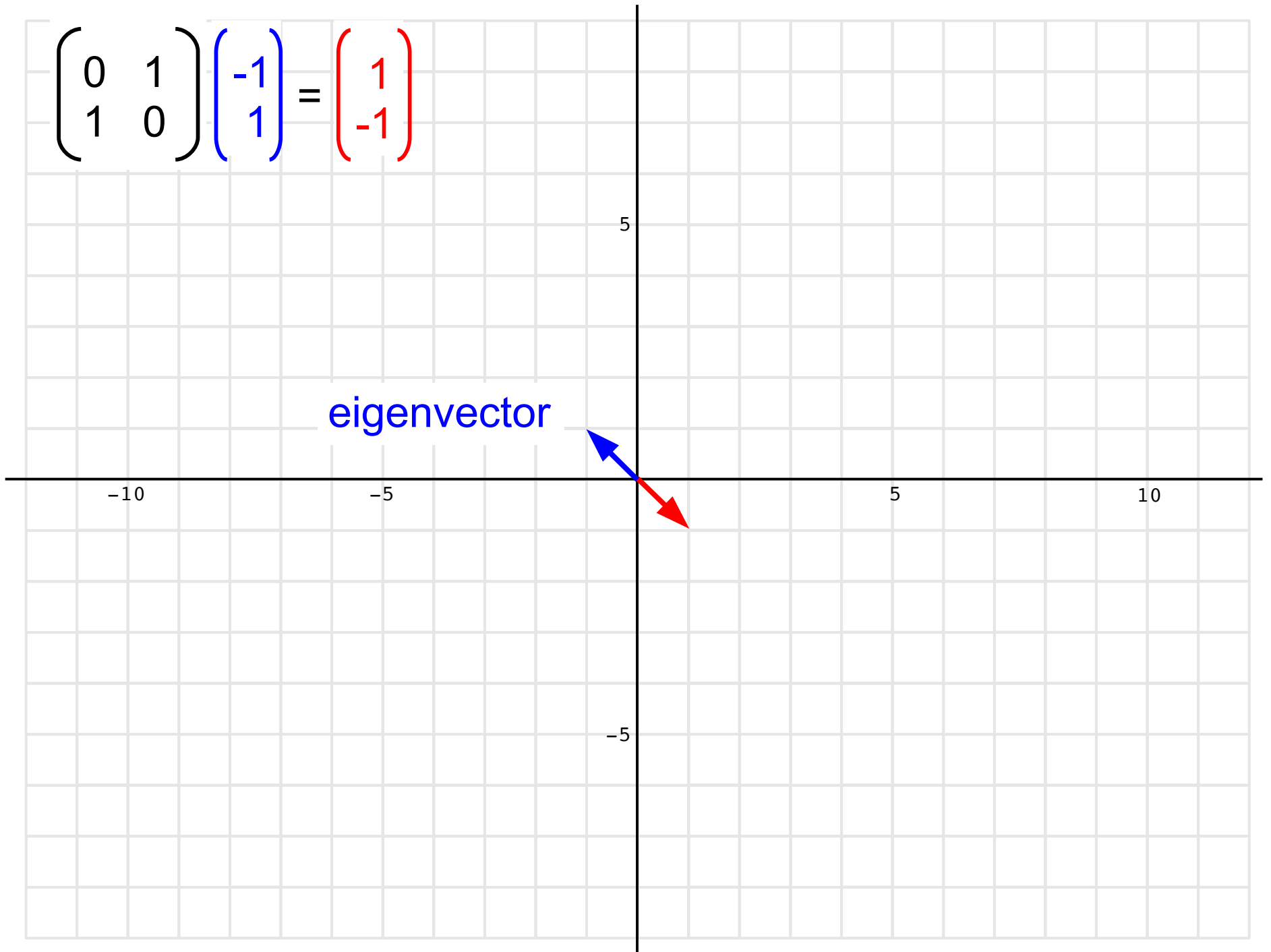


$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$



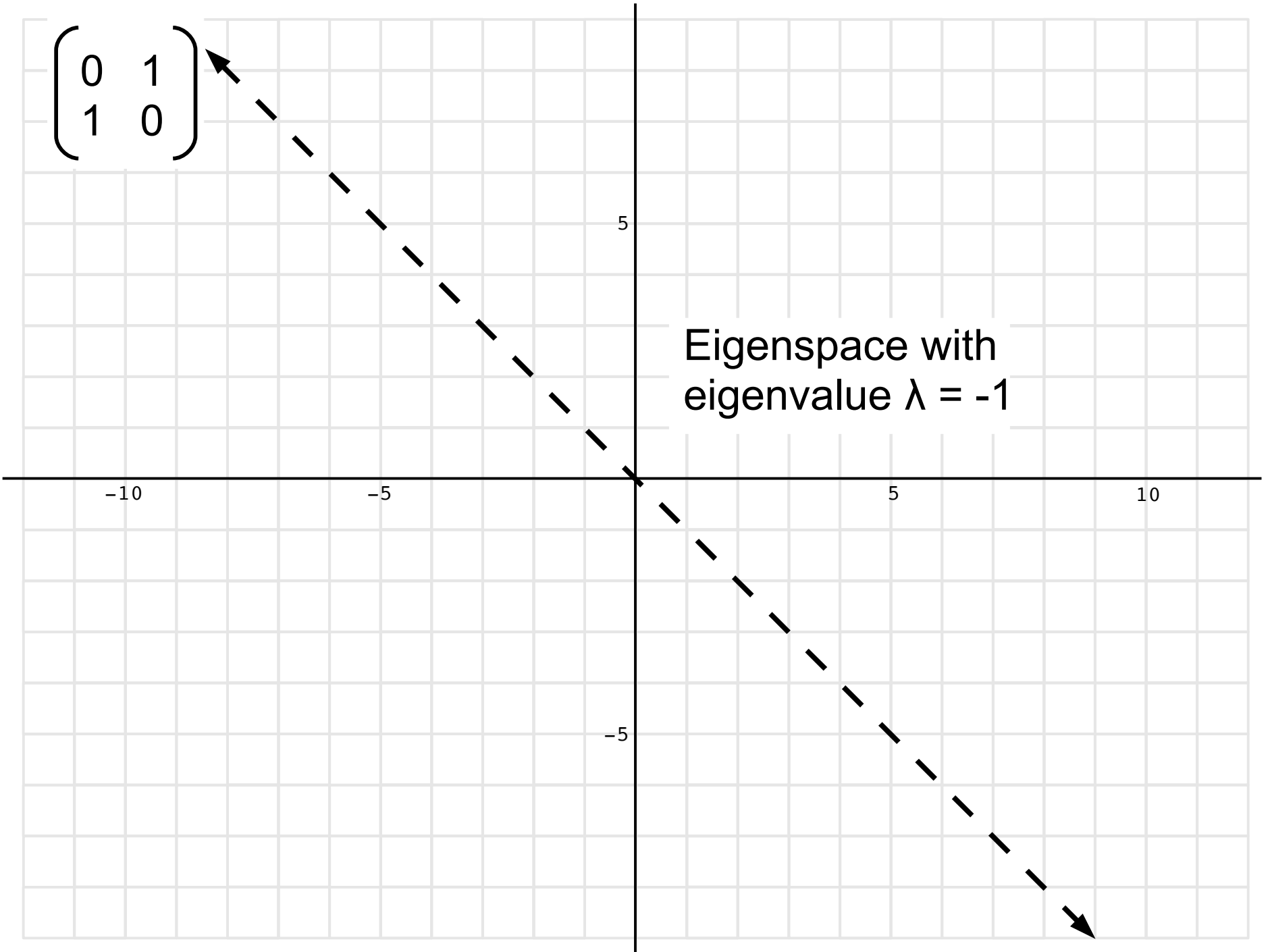
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

eigenvector



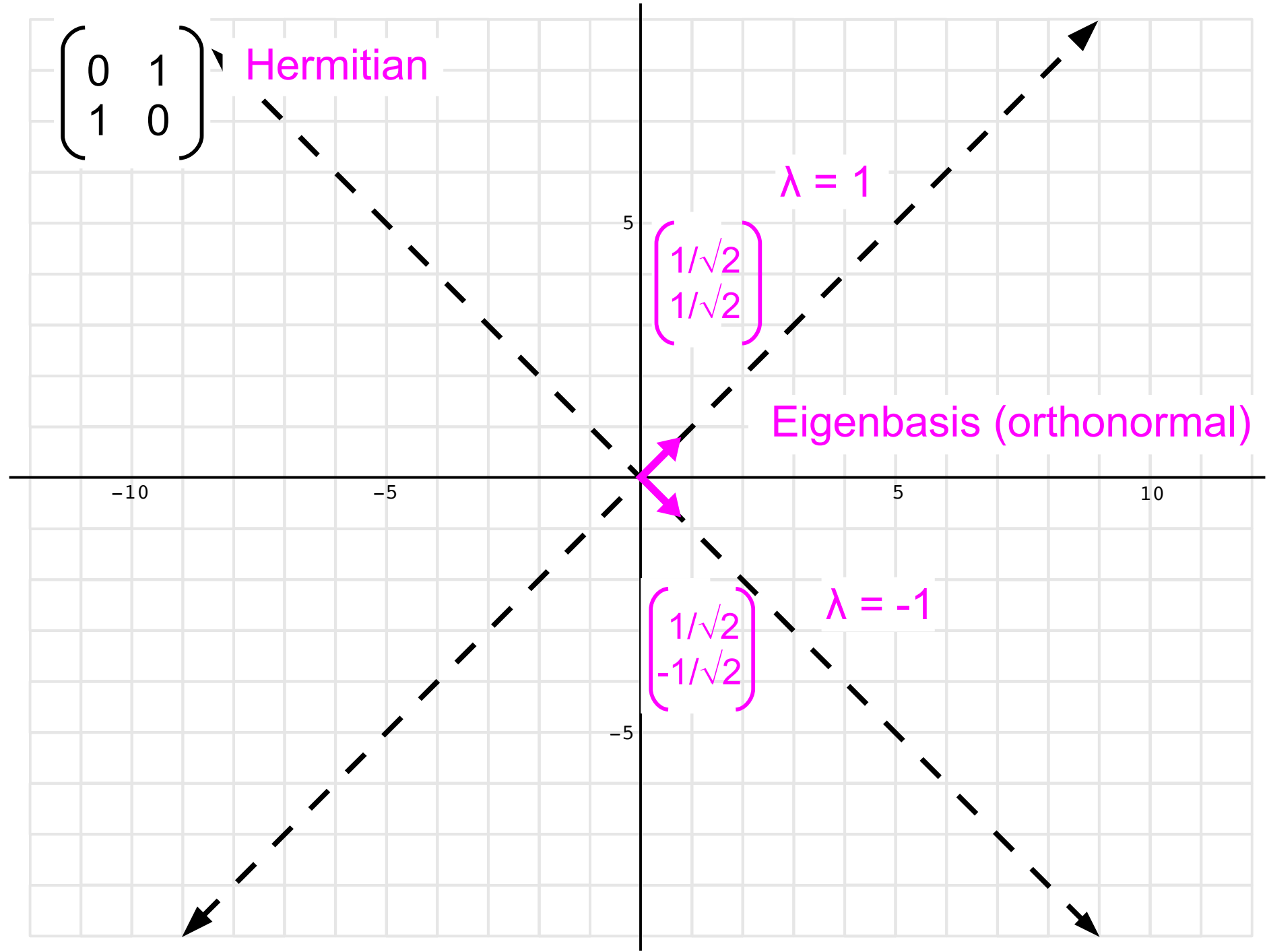
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Eigenspace with
eigenvalue $\lambda = -1$



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hermitian



$\lambda = 1$

$$\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

Eigenbasis (orthonormal)

$\lambda = -1$

$$\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$