Assignment 18

Due by class time Tuesday, November 22

1. What if we use $|1\rangle \otimes |0\rangle$ instead of $|0\rangle \otimes |1\rangle$ as the inputs to Deutsch's algorithm? Does the algorithm still distinguish constant from balanced functions? Explain.

2. What if we use $|1\rangle \otimes |1\rangle$ instead of $|0\rangle \otimes |1\rangle$ as the inputs to Deutsch's algorithm? Does the algorithm still distinguish constant from balanced functions? Explain.

3. Consider a function $f(\mathbf{x})$ that takes 128-bit binary strings as input and returns a single bit as output. There are a total of 2^{128} possible input strings $\mathbf{x}$. f is *constant* if $f(\mathbf{x})$ is the same value (0 or 1) for every $\mathbf{x}$. f is *balanced* if $f(\mathbf{x})$ gives 0 for exactly half of the inputs, and 1 for the other half. There are many functions that are neither constant nor balanced, but suppose that we know for sure that f is one or the other.

   (a) If we evaluate f on successive inputs in some fixed order (so that we never evaluate f more than once on the same input), using a classical computer, what is the *minimum* number of evaluations needed, in the best case, to determine whether f is balanced or constant? Explain.

   (b) What is the *maximum* number of evaluations needed, in the worst case, using a classical computer? Explain.

4. Read section 6.2 of *Quantum Computing for Computer Scientists* (pages 179–187).