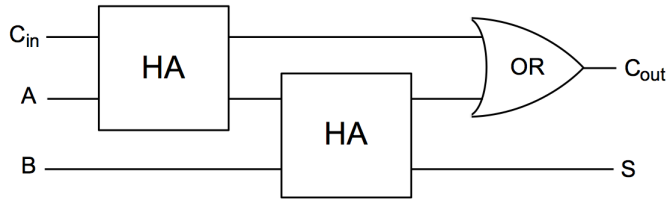# Assignment 14

Due by class time Tuesday, November 8

1. A logic diagram and truth table for a Full-Adder circuit (FA) is given below:

| Cin	A	B		Cout	S
0	0	0		0	0
0	0	1		0	1
0	1	0		0	1
0	1	1		1	0
1	0	0		0	1
1	0	1		1	0
1	1	0		1	0
1	1	1		1	1

   (a) Find the corresponding matrix for FA.

   (b) Write an equivalent expression for FA that combines the matrices HA, $IDEN$, and OR using matrix multiplication and the tensor product (where HA is the matrix for the Half-Adder circuit from the previous assignment, and $IDEN$ is the 2×2 identity matrix).

2. The symbol for the controlled-NOT gate (CNOT) and its truth table are shown below, with the output $y' = x \oplus y$:
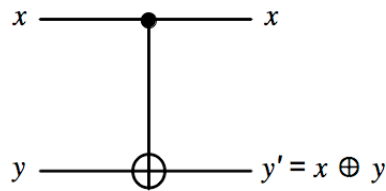
x	y		x	y'
0	0		0	0
0	1		0	1
1	0		1	1
1	1		1	0

   (a) Show how to construct a COPY gate from a single CNOT gate.

   (b) A SWAP gate takes two input bits and simply swaps their order, as shown in the diagram and truth table below. Show how to construct a SWAP gate from three CNOT gates. Hint: the XOR identities $z \oplus z = 0$ and $z \oplus 0 = z$ may be useful.
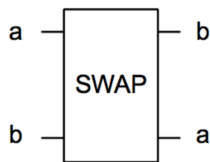
a	b		b	a
0	0		0	0
0	1		1	0
1	0		0	1
1	1		1	1

3. The symbol for a Toffoli gate and its corresponding truth table are shown below, where the output $z' = z \oplus (x \wedge y)$:
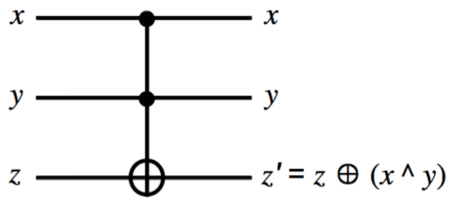
x	y	z			x	y	z'
0	0	0			0	0	0
0	0	1			0	0	1
0	1	0			0	1	0
0	1	1			0	1	1
1	0	0			1	0	0
1	0	1			1	0	1
1	1	0			1	1	1
1	1	1			1	1	0

(a) Show how to construct a NAND gate using a single Toffoli gate.

(b) Show how to construct a NOR gate using three Toffoli gates.

(c) Show how to construct an OR gate using three Toffoli gates.

(d) Show how to construct an OR gate using only two Toffoli gates.