Assignment 13

Due by class time Thursday, November 3

0. If you haven't already done so, finish reading section 5.2 of the textbook (pages 144–151).

1. Using the truth tables shown below for the operations XOR and XNOR, find binary matrices that correspond to an XOR gate and an XNOR gate. The output of a XOR b is 1 exactly when $a \neq b$, and a XNOR b is 1 exactly when $a = b$.

a	b	XOR
0	0	0
0	1	1
1	0	1
1	1	0

a	b	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

2. Below is the truth table for a COPY gate, which takes a single bit a as input and outputs two copies of the bit. Find a matrix that corresponds to this gate.

a	out_1	out_2
0	0	0
1	1	1

3. Below is the truth table for a SWAP gate, which takes two input bits a and b, and outputs the bits in the reverse order. Find a matrix that corresponds to this gate.

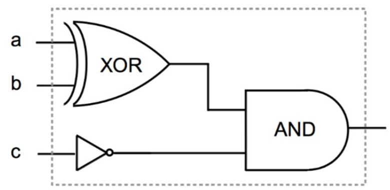
a	b	out_1	out_2
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

4. Below is the truth table for a 1-bit "half adder" circuit. A half adder takes two input bits a and b, and produces two output bits: the 1-bit sum S of a and b in binary, and the carry bit C_{out}. Find the corresponding matrix for this truth table.

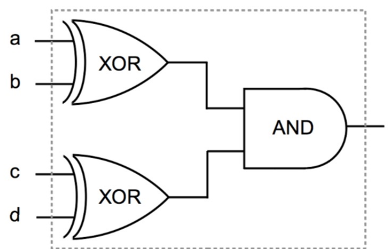
a	b	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

5. Following Example 5.2.2 on page 149 of the textbook, write an expression that combines the matrices for *NOT*, *AND*, *OR*, *XOR*, or *IDEN* using matrix multiplication ($\star$) and the tensor product ($\otimes$) for each of the logic circuits below, where *IDEN* is the 2×2 identity matrix. In each case, also show the final matrix generated by your expression, and give its dimensions. Make sure that the dimensions correspond appropriately to the number of inputs and outputs for each circuit.

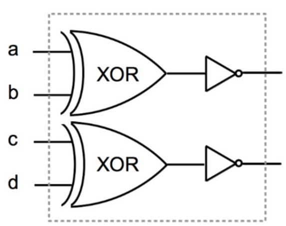
(a)



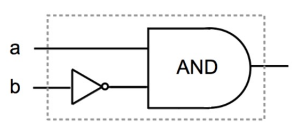
(b)



(c)



(d)



(e)