Assignment 11

Due by class time Thursday, October 27

Note: You should show your work for each of these problems, but you can use WolframAlpha.com to double check your matrix computations. For example, the matrix multiplication $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix}$ can be entered as `{1,2}.{{3,4},{5,6}}.{{7},{8}}`

In his paper, "Is the moon there when nobody looks?", David Mermin imagines a particle having the observable properties *1-color*, *2-color*, and *3-color*, which we can represent by the hermitian matrices C_1, C_2, and C_3, along with their corresponding eigenbasis vectors shown below:

$$C_1 \text{ (1-color)} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad |R_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |G_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_2 \text{ (2-color)} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -0.5 & 0.866 \\ 0.866 & 0.5 \end{bmatrix} \qquad |R_2\rangle = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \qquad |G_2\rangle = \begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$C_3 \text{ (3-color)} = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -0.5 & -0.866 \\ -0.866 & 0.5 \end{bmatrix} \qquad |R_3\rangle = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \qquad |G_3\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

The observed value of a color property can be either red or green. The eigenvalue $+1$ is associated with the "red" eigenstates $|R_1\rangle$, $|R_2\rangle$, and $|R_3\rangle$, and the eigenvalue -1 is associated with the "green" eigenstates $|G_1\rangle$, $|G_2\rangle$, and $|G_3\rangle$. For example, if a detector measures a particle's 1-color and flashes red as a result, the particle will be in state $|R_1\rangle$ after the measurement.

1. Suppose a particle is in state $|R_1\rangle$ (its 1-color is red) when it arrives at a detector whose switch is set to 2 (that is, the detector is set to measure the 2-color property of the particle). What is the probability that the detector will flash red? What is the probability that the detector will flash green? Show how the probability is calculated in each case.

2. Suppose that the detector had measured the particle's 3-color instead of its 2-color upon arrival. What would the probabilities of flashing red or green have been in that case? Show your calculation.

3. What is the *expected value* (or mean) of the 2-color of a particle arriving in state $|R_1\rangle$? Show your calculation.

4. What is the *expected value* (or mean) of the 3-color of a particle arriving in state $|G_3\rangle$? Show your calculation.

5. (a) In question 3, you calculated the mean value (let's call it m) of 2-color in state $|R_1\rangle$. We can define a new observable, called the "mean-adjusted" or "demeaned" 2-color in state $|R_1\rangle$, by simply subtracting m from the diagonal entries of C_2. This new observable, denoted $\Delta_{R_1}(C_2)$, represents the *raw deviation* from the mean value of 2-color in state $|R_1\rangle$. Show its matrix. Is it still hermitian?

   (b) Take your matrix from part (a) and square it. That is, calculate $\Delta_{R_1}(C_2) \star \Delta_{R_1}(C_2)$. This new observable represents the *squared deviation* from the mean value of 2-color in state $|R_1\rangle$. Is it still hermitian?

   (c) Now calculate the *expected value* of your observable $\Delta_{R_1}(C_2)\star\Delta_{R_1}(C_2)$ in state $|R_1\rangle$. This is the *variance* of 2-color in $|R_1\rangle$, and is denoted $\mathrm{Var}_{R_1}(C_2)$. Intuitively, this number represents the average "spread" of the values one would obtain by measuring 2-color in $|R_1\rangle$ over and over again. (Or more precisely, the average *squared deviation from the mean value* of measuring 2-color in $|R_1\rangle$ over and over.)

   (d) In general, the variance of an observable in some states may be high, meaning that a measurement will exhibit significant uncertainty, while in other states, the variance may be low, or possibly even zero, meaning that there is little to no uncertainty in the result. Put another way, the higher the variance, the "fuzzier" the measurement, and the lower the variance, the "sharper" the measurement. Are there any states for which the variance of 2-color is precisely 0, meaning no uncertainty at all in the outcome? If so, give an example of such a state. What about for 1-color or 3-color?