Assignment 10

Due by class time Thursday, October 20

As a reminder, here are the three spin operators with their associated eigenvalues and eigenvectors:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \lambda_1 = +1,\ |right\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \qquad \lambda_2 = -1,\ |left\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \lambda_1 = +1,\ |in\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} \qquad \lambda_2 = -1,\ |out\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \lambda_1 = +1,\ |up\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \lambda_2 = -1,\ |down\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

1. Suppose we measure a quantum particle's spin in the Y direction and the result is $+1$. The particle will then be definitively in state $|in\rangle = [\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}]^T$. We can express this state in terms of each of the three spin operator eigenbases by writing it as a linear combination of the basis states. Calculate the corresponding amplitudes for each eigenbasis and fill in the blanks below accordingly, showing your work.

   $|in\rangle =$ ____________ $|right\rangle$ + ____________ $|left\rangle$

   $|in\rangle =$ ____________ $|in\rangle$ + ____________ $|out\rangle$

   $|in\rangle =$ ____________ $|up\rangle$ + ____________ $|down\rangle$

2. Suppose our particle is in state $|in\rangle$ and we measure the observable X (spin in the right/left direction). What is the probability of obtaining $+1$? What is the probability of obtaining -1? Justify your answers.

3. Suppose our particle is in state $|in\rangle$ and we measure the observable Y. What is the probability of obtaining $+1$? What is the probability of obtaining -1? Justify your answers.

4. Suppose we prepare our particle in state $|down\rangle$ and then measure the observable Ω, which has the following eigenvalues and orthonormal eigenbasis:

   $$\Omega = \begin{bmatrix} 3 & 1+i \\ 1-i & 2 \end{bmatrix} \qquad \lambda_1 = 4,\ |e_1\rangle = \begin{bmatrix} \frac{1+i}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \qquad \lambda_2 = 1,\ |e_2\rangle = \begin{bmatrix} \frac{-1-i}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}$$

   What is the probability of obtaining 4 as the result of the measurement? What is the probability of obtaining 1? What is the probability of obtaining 2.5 (the average of 4 and 1)? Justify your answers.

5. What is the expected (mean) value of repeatedly measuring the observable Ω from the previous exercise in state $|down\rangle$? Calculate it as $\langle down|\,\Omega\,|down\rangle$. Then calculate it again as the weighted average of the eigenvalues λ_1 and λ_2. Show your work. Do your two answers agree? (They should.)

6. Consider the observable Ω below, with its associated eigenvalues and eigenbasis vectors:

$$\Omega = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \qquad \lambda_1 = +1, \quad |e_1\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \qquad \lambda_2 = -1, \quad |e_2\rangle = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

Suppose we prepare the system in state $|\psi\rangle = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{bmatrix}$

(a) If we measure Ω in state $|\psi\rangle$, what is the probability p_1 of obtaining $+1$ as the outcome of the measurement? What is the probability p_2 of obtaining -1?

(b) Calculate the expected (mean) value $\langle\psi|\,\Omega\,|\psi\rangle$ of repeatedly observing Ω in state $|\psi\rangle$.

(c) Calculate the weighted average $p_1\lambda_1 + p_2\lambda_2$, where p_1 and p_2 are the probabilities you calculated in part (a). Does your value agree with your answer for part (b)? (It should.)

7. Read the article "Is the moon there when nobody looks?" by David Mermin, which I passed out in class today. We'll discuss this article in class next week.