Assignment 9

Due by class time Thursday, October 13

1. Write a Python program called **showVectors(*n*)** that takes an integer $n \geq 1$ as input and prints out all 2^n binary vectors of dimension n, *in binary order*, one vector per line. For example, if $n = 2$, your program should print out [0, 0], [0, 1], [1, 0], and [1, 1], in that order. For example:

```
>>> showVectors(2)
[0, 0]
[0, 1]
[1, 0]
[1, 1]

>>> showVectors(3)
[0, 0, 0]
[0, 0, 1]
[0, 1, 0]
[0, 1, 1]
[1, 0, 0]
[1, 0, 1]
[1, 1, 0]
[1, 1, 1]
```

2. Now let the dimensionality of vector space A be any integer $m \geq 1$ and the dimensionality of vector space B be any integer $n \geq 1$. Write a program called **showTensors(*m, n, mode*)** that takes m and n as input parameters and does what we did in class. That is, your program should determine all of the distinct *separable* and *entangled* (non-separable) tensor product vectors in $A \otimes B$, and print them out, one vector per line, with the basis vectors highlighted in some way. It should also report the dimensionality of $A \otimes B$, the total number of vectors in $A \otimes B$, the number of separable vectors, and the number of entangled vectors.

   The third input parameter (called *mode*) should be the string `"brief"` or `"all"`, and should control the amount of output produced by the program. When *mode* is `"brief"`, the individual vectors should not be printed — only the dimension of $A \otimes B$, the total number of vectors in $A \otimes B$, and the total number of separable and entangled vectors, should be reported. When *mode* is `"all"`, all information should be reported, including the individual vectors.

   IMPORTANT: You should include comments at the top of your code clearly explaining how to run your programs, and a separate demo program that tests **showVectors** and **showTensors** on some sample inputs.

   For example, the next page shows sample output for the case of $m = 3$ and $n = 2$:

```
>>> showTensors(3, 2, "all")
Tensor product space is 6-dimensional
22 distinct separable tensor product vectors out of 64 possible
[0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 1] <-- basis vector
[0, 0, 0, 0, 1, 0] <-- basis vector
[0, 0, 0, 0, 1, 1]
        ...
[1, 1, 1, 1, 1, 1]

42 entangled (non-separable) tensor product vectors out of 64 possible
[0, 0, 0, 1, 1, 0]
[0, 0, 0, 1, 1, 1]
[0, 0, 1, 0, 0, 1]
[0, 0, 1, 0, 1, 1]
        ...
[1, 1, 1, 1, 1, 0]

>>> showTensors(3, 2, "brief")
Tensor product space is 6-dimensional
22 distinct separable tensor product vectors out of 64 possible
42 entangled (non-separable) tensor product vectors out of 64 possible
```

3. What happens to the relative proportion of separable and entangled vectors in $A \otimes B$ as the dimensionality of A and B increases? Use your program to fill in the table below for the values of m and n shown. Beware of larger values!

$m \otimes n$	dimensionality	total # of vectors	# of separable	# of entangled
$1 \otimes 2$				
$2 \otimes 2$				
$3 \otimes 2$	6	64	22	42
$2 \otimes 3$				
$3 \otimes 3$				
$4 \otimes 4$				
$4 \otimes 5$				
$5 \otimes 5$				