Assignment 8

Due by class time Thursday, October 6

1. Calculate the tensor products $\begin{bmatrix}4\\2\\1\end{bmatrix} \otimes \begin{bmatrix}3\\-1\\0\\2\end{bmatrix}$ and $\begin{bmatrix}3\\-1\\0\\2\end{bmatrix} \otimes \begin{bmatrix}4\\2\\1\end{bmatrix}$.

   In general, does $\otimes$ commute?

2. The standard RGB triplet for orange is (255, 165, 0), which, when normalized to length 1 in our 3-dimensional "RGB spheramid" vector space, is (approximately) the vector $[0.84, 0.54, 0]^T$.

   (a) Give the 9-dimensional tensor product state vector representing both me and you wearing orange shirts.

   (b) Give the tensor product state vector representing me wearing an orange shirt and you wearing a blue shirt.

   (c) Give the tensor product state vector representing me wearing a blue shirt and you wearing an orange shirt.

3. Suppose our 9-dimensional tensor product state vector is $[0, 0, 0, \frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, 0]^T$. What are the colors of my shirt and your shirt? Justify your answer by showing how this state vector can be written as the tensor product of two 3-dimensional "RGB spheramid" state vectors.

4. Show that in general, if the vectors $V_1 = [a_0, a_1, a_2]^T$ and $V_2 = [b_0, b_1, b_2]^T$ are both of length 1, then their tensor product $V_1 \otimes V_2$ will also be of length 1.