Assignment 7

Due by class time Tuesday, October 4

0. More practice exercises from the book (NOT to turn in; answers are in Appendix B):

   (a) Exercise 2.5.1 (page 62) — eigenvalues

   (b) Exercise 2.6.1 (page 63) — hermitian matrices

1. Consider the matrix $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ and the vectors $|E_1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$ and $|E_2\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{bmatrix}$.

   (a) Show that $|E_1\rangle$ and $|E_2\rangle$ are both eigenvectors of Y.

   (b) What are their associated eigenvalues?

   (c) Using the inner product, show that these vectors form an *orthonormal basis*. That is, show that they are perpendicular to each other, and that the length of each vector is 1.

2. For what values of θ, if any, is the matrix below *hermitian*? Explain.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Show that the above matrix is *unitary* for all values of θ. Hint: remember that $\sin^2\theta + \cos^2\theta = 1$.

4. The standard RGB triplet for orange is (255, 165, 0). What would the corresponding vector for orange be in our "RGB spheramid" vector space? Hint: just normalize it to length 1.

5. Imagine a solid 3-dimensional sphere centered on $(0, 0, 0)$. The portion of the sphere containing points with only non-negative coordinate values (*i.e.*, $x \geq 0$ and $y \geq 0$ and $z \geq 0$) is $\frac{1}{8}$ of the entire sphere. What is the analogous fraction for a 9-dimensional hypersphere? For a D-dimensional hypersphere?