Assignment 6

Due by class time Thursday, September 29

0. More practice exercises from the book (NOT to turn in; answers are in Appendix B)

   (a) Exercise 2.4.1 (page 55) — inner product
   (b) Exercise 2.4.5 (page 56) — norm of a vector
   (c) Exercise 2.4.7 (page 57) — norm of a vector

1. Calculate the inner product $\langle V_1 \,|\, V_2 \rangle$ of the vectors $V_1 = \begin{bmatrix} 2+i \\ 1+3i \\ 4 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 2+i \\ -i \\ -2+i \end{bmatrix}$.

2. Calculate the norm $\|V\|$ of the vector $V = \begin{bmatrix} 1+i \\ 3-2i \\ 2 \end{bmatrix}$.

3. Now calculate $\sqrt{\langle V \,|\, V \rangle}$ where V is the vector in the previous exercise, and verify that the result is equal to the norm of V.

4. In class, we expressed the vector $V = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$ in two different ways, one in terms of the basis vectors $|A_1\rangle$ and $|A_2\rangle$:

   $$V = 0.5\,|A_1\rangle + 0.866\,|A_2\rangle \qquad \text{where } |A_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |A_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   and the other in terms of the basis vectors $|B_1\rangle$ and $|B_2\rangle$:

   $$V = 0.966\,|B_1\rangle - 0.259\,|B_2\rangle \qquad \text{where } |B_1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \text{ and } |B_2\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

   Consider the basis vectors $|C_1\rangle = [-0.809, 0.588]^T$ and $|C_2\rangle = [0.588, 0.809]^T$, where the values 0.809 and 0.588 are approximations of $\cos(\frac{9\pi}{5})$ and $\sin(\frac{4\pi}{5})$, respectively.

   (a) Show that this basis is *orthonormal*. That is, use the inner product to show that the norm of each basis vector is 1, and that the vectors are orthogonal. Hint: two vectors are orthogonal if and only if their inner product is 0.

   (b) Express V in terms of this basis, that is, as a linear combination of $|C_1\rangle$ and $|C_2\rangle$. Show your work.