

Assignment 4

Due by class time Thursday, September 22

1. What is the length of the diagonal of a 2-dimensional “cube” (otherwise known as a “square”) with sides of length 1? What is the length of the diagonal of a 3-dimensional cube with sides of length 1? Of a 4-dimensional cube? Of a 5-dimensional cube? Of a D -dimensional cube?
2. For a D -dimensional cube, starting from any vertex, how many vertices are reachable in one step by following an edge? How many steps does it take to reach the diametrically opposite vertex?
3. How many vertices does a D -dimensional cube have?
4. How many faces does a D -dimensional cube have? (The “faces” of a cube are those regions that together define the boundary between points “inside” and points “outside” of the cube.) What is the dimensionality of each face?
5. When we project a D -dimensional cube into a lower $(D - 1)$ -dimensional space, how many of its faces get distorted by the projection, meaning that the face’s edges are no longer all perpendicular to each other, and how many remain undistorted?

Extra Credit (optional)

6. How many edges does a 4-dimensional cube have? How many does a 5-dimensional cube have? Give a formula for calculating the number of edges for any dimension D , expressed in terms of the number of edges of a $(D - 1)$ -dimensional cube. Can you come up with a general formula for the number of edges that depends only on the variable D ?