Assignment 1

Due by class time Tuesday, September 13

For these problems, keep in mind the following mathematical relationships between the four "component parts" of a complex number, which we are calling its *real part* (a), its *imaginary part* (b), its *magnitude* (ρ), and its *phase* (θ):

$$a = \rho \cos\theta \qquad b = \rho \sin\theta \qquad \rho = \sqrt{a^2 + b^2} \qquad \theta = \tan^{-1}\frac{b}{a}$$

We can represent any complex number z either in terms of a, b, i, addition, and multiplication, as $z = a + bi$, or equally well in terms of ρ, θ, e, i, multiplication, and exponentiation as $z = \rho e^{\theta i}$.

1. The complex numbers below are written in the Cartesian form $a + bi$. Rewrite each one in the exponential form $\rho e^{\theta i}$, with the phase θ expressed in radians from 0 to 2π. Show your work.

   (a) $3 + 4i$
   (b) $-3 + 4i$
   (c) $-3 - 4i$
   (d) $3 - 4i$
   (e) $4i$
   (f) 7
   (g) $-i$
   (h) $-2 + i$

2. The complex numbers below are written in the exponential form $\rho e^{\theta i}$, with the phase θ in radians. Rewrite each one in the Cartesian form $a + bi$. Show your work.

   (a) $4e^{\pi i/3}$
   (b) $2\sqrt{3}\, e^{4\pi i/3}$
   (c) $5e^{2\pi i}$
   (d) $5e^{3\pi i/2}$
   (e) $3e^{\pi i/2}$
   (f) e^{-i}
   (g) $\sqrt{2}\, e^{\pi i/4}$
   (h) $e^{-2\pi i/3}$

3. Calculate $(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)^{100}$. Show your work. Hint: don't do it the hard way!