Propositional Calculus

Atoms:

P   Q   R   P′   Q′   R′   P″   Q″   R″   P‴   *… etc.*

Six other symbols:

<   >   ^   v   ~   ⊃

Well-formed strings:

~P   < P ^ Q >   < Q v R >   < P' ⊃ Q' >

< ~P ^ < P ^ Q > >   < < Q v R > ⊃ < P' ⊃ Q' > >

Which Strings Are Well-Formed?

<P>

<~P>

<P ^ Q ^ R>

<P ^ Q>

<<P ^ Q> ^ <Q ~^ P>>

<P ^ ~P>

<<P v <Q ⊃ R>> ^ <~P v ~R'>>>

<P ^ Q> ^ <Q ^ P>

Propositional Calculus Rules

- Joining: from **A** and **B** we can make **< A ∧ B >**
- Separation: from **< A ∧ B >** we can make both **A** and **B**
- Double-tilde: **~~** can be inserted or deleted as long as the resulting string is well-formed
- Fantasy: if we can conclude **B** by starting from an assumption of **A**, then we can make **< A** ⊃ **B >**
- Carry-over: in a fantasy, we can use anything already made before
- Detachment: from **A** and **< A** ⊃ **B >** we can make **B**
- Contrapositive: **< A** ⊃ **B >** is interchangeable with **< ~B** ⊃ **~A >**
- DeMorgan: **< ~A ∧ ~B >** is interchangeable with **~<A v B >**
- Switcheroo: **< A v B >** is interchangeable with **< ~A** ⊃ **B >**

Translating English Into Symbols

H = *it is hot*
S = *it is sunny*

"it is hot and sunny"	< H ^ S >
"it is not sunny"	~S
"it is not hot, but it is sunny"	< ~H ^ S >
"it is neither hot nor sunny"	< ~H ^ ~S >
"if it is sunny, then it is hot"	< S ⊃ H >
"it's either hot, or it isn't"	< H v ~H >

Translating English Into Symbols

B = *the bus was late*
W = *my watch was fast*

“the bus was late or my watch was fast”

< B v W >

“it wasn't that the bus was late or my watch was fast”

~< B v W >

Apply De Morgan's Rule: < ~B ^ ~W >

“the bus was not late and my watch was not fast”

Translating English Into Symbols

W = *you get to work on time*
F = *you are fired*

"if you do not get to work on time, then you are fired"

$< \sim W \supset F >$

Apply Switcheroo Rule:

$< W \lor F >$

"either you get to work on time or you are fired"

Translating English Into Symbols

L = *you can swim across the lake*
B = *you can swim to the boat*

"if you can swim across the lake, then you can swim to the boat"

$< L \supset B >$

Apply Contrapositive Rule:

$< \sim B \supset \sim L >$

"if you cannot swim to the boat, then you cannot swim across the lake"

Translating English Into Symbols

R = *my Rolls Royce is in the repair shop*
C = *I can get to class*

"if my Rolls is in the repair shop, then I cannot get to class"

$< R \supset \sim C >$

Negation:

$\sim< R \supset \sim C >$

"it is not the case that if my Rolls is in the repair shop, then I cannot get to class"

Applying The Rules

“it is not the case that if my Rolls is in the repair shop, then I cannot get to class”

	~< R ⊃ ~C >
Contrapositive:	~< ~~C ⊃ ~R >
Switcheroo:	~< ~C v ~R >
DeMorgan:	< ~~C ^ ~~R >
Double-tilde:	< C ^ ~~R >
Double-tilde:	< C ^ R >

“I can get to class and my Rolls is in the repair shop”

In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note, he wrote that he had hidden treasure somewhere on the property. He listed five true statements and challenged the reader to use them to figure out the location of the treasure:

(a) If this house is next to a lake, then the treasure is not in the kitchen.

(b) If the tree in the front yard is an elm, then the treasure is in the kitchen.

(c) This house is next to a lake.

(d) The tree in the front yard is an elm or the treasure is buried under the flagpole.

(e) If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

L = house is next to a lake
K = treasure is in the kitchen
E = tree in front yard is an elm
F = treasure is buried under the flagpole
O = tree in back yard is an oak
G = treasure is in the garage

(a) < L ⊃ ~K >	premise
(b) < E ⊃ K >	premise
(c) L	premise
(d) < E v F >	premise
(e) < O ⊃ G >	premise

(1) ~K	detachment: (a), (c)
(2) < ~K ⊃ ~E >	contrapositive: (b)
(3) ~E	detachment: (1), (2)
(4) < ~E ⊃ F >	switcheroo: (d)
(5) F	detachment: (3), (4)

So the treasure is buried under the flagpole!

You are about to leave for school in the morning and discover that you don't have your glasses. You know the following statements are true:

(a) If I was reading the newspaper in the living room, then my glasses are on the coffee table.

(b) If my glasses are on the kitchen table, then I saw them at breakfast.

(c) If I was reading my book in bed, then my glasses are on the nightstand.

(d) I was reading the newspaper either in the living room or in the kitchen.

(e) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

(f) I did not see my glasses at breakfast.

Where are the glasses?

A = reading newspaper in living room
B = reading newspaper in kitchen
C = reading book in bed
D = glasses on coffee table
E = glasses on kitchen table
F = glasses on nightstand
G = saw glasses at breakfast

(a)	< A ⊃ D >	premise
(b)	< E ⊃ G >	premise
(c)	< C ⊃ F >	premise
(d)	< A v B >	premise
(e)	< B ⊃ E >	premise
(f)	~G	premise

(1)	< ~G ⊃ ~E >	contrapositive: (b)
(2)	~E	detachment: (f), (1)
(3)	< ~E ⊃ ~B >	contrapositive: (e)
(4)	~B	detachment: (2), (3)
(5)	< ~A ⊃ B >	switcheroo: (d)
(6)	< ~B ⊃ ~~A >	contrapositive: (5)
(7)	< ~B ⊃ A >	double-tilde: (6)
(8)	A	detachment: (4), (7)
(9)	D	detachment: (a), (8)

So the glasses are on the coffee table!