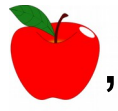
Logical Paradoxes

- **Epimenides paradox:** "This statement is false"
- **Russell's paradox**
  - Sets are collections of **things**

    {  ,  , "apple" } { 1, 5, 42 } { }
  - A set is also a **thing** in its own right, so sets can contain other sets

    { 37, 0, { 1, 5, 42 }, 19, { }, 88 }
  - A set can even contain **itself** (Example: the set of all sets)
  - R: the set of **all sets that don't contain themselves**
- **Does R contain itself?**

  No? Then it **must** contain R, by definition!

  Yes? Then it **can't** contain R, by definition!

The Theory of Types

- Objects:   “apple” 5 42
- Level 1 sets: {  , 42, “apple” } { 1, 2, 3 } { }
- Level 2 sets: { “apple”, { 1, 2, 3 },  } { { } }
- Level 3 sets: { 42, { { } }, { “apple”, { 1, 2, 3 },  } }
- … and so on …
- No self-reference (or self-membership) allowed!

Logical Paradoxes

- **Barber paradox**
  - The town of Smoothville has just one barber
  - Nobody in this town has a beard
  - Every man either shaves himself, or goes to the barber
  - The barber shaves only men who don't shave themselves

- Who shaves the barber?
  - He can't shave himself, because he only shaves men who don't shave themselves
  - He can't let the barber do it, because he's the barber

- Too contrived to really be a genuine paradox

Logical Paradoxes

- **Berry paradox**
  - Some positive numbers can be described in a **single word**:

    “zero”, “nine”, “eighty”, etc.
  - Many can be described in **eleven words or less**:

    “two hundred”, “a googol”, “the product of five and six”, “the smallest prime”, “the number halfway between fifty and a hundred”
  - The number of phrases of eleven words or less is **finite**
  - So there must be many, many positive numbers that **cannot be described** in eleven words or less
  - Of these, which one is the smallest?
  - **“the smallest positive number not describable in eleven words or less”**