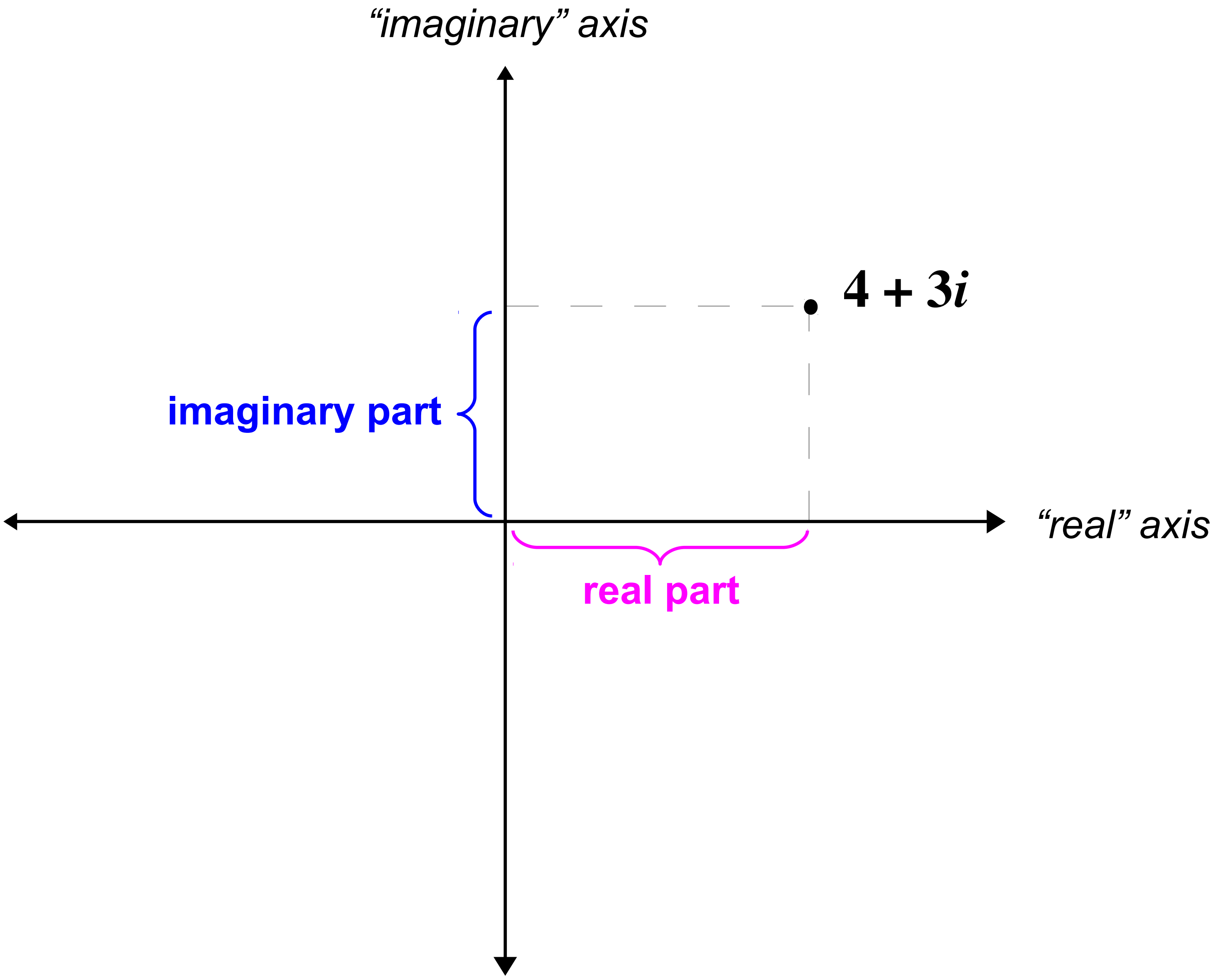
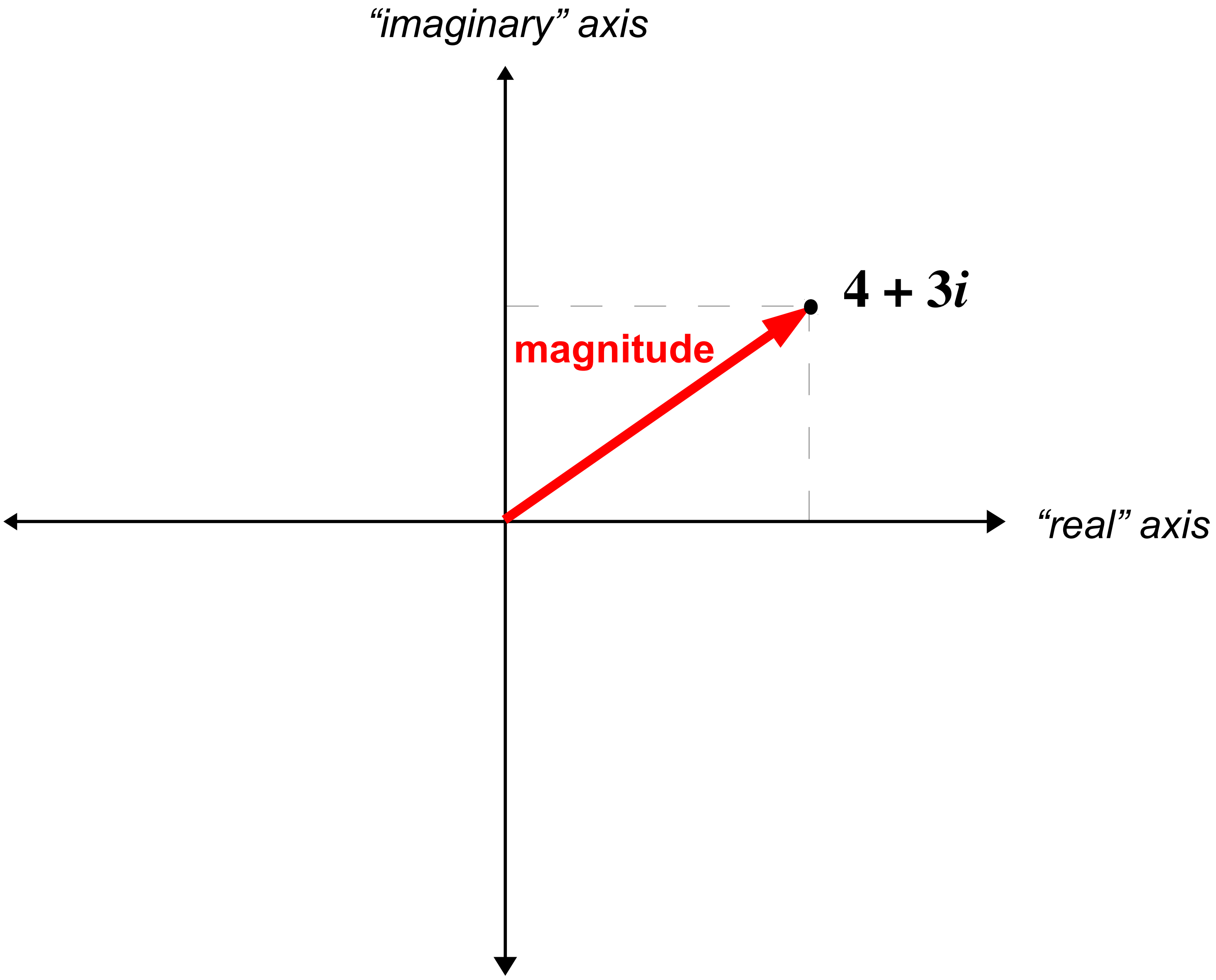
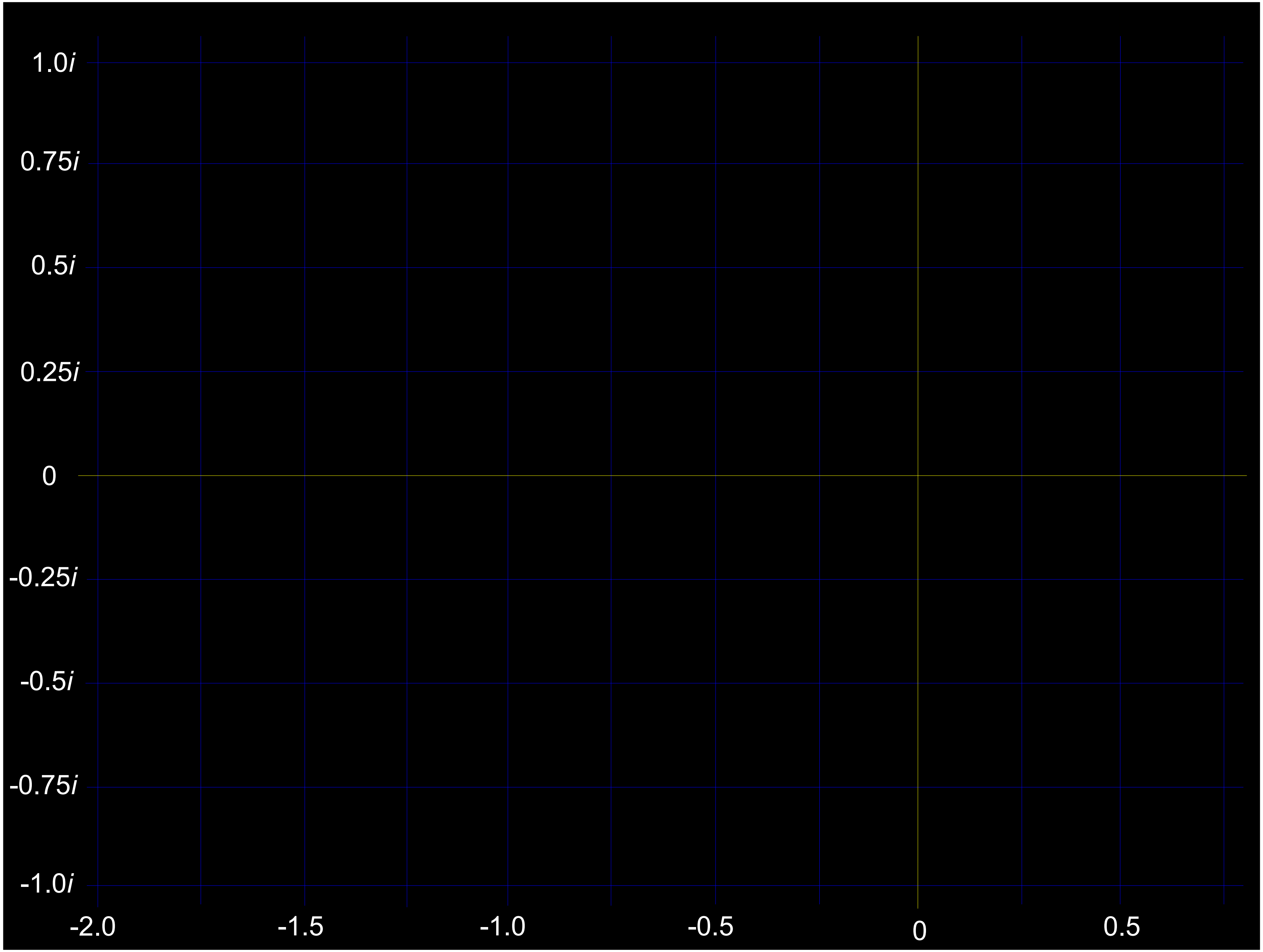
Complex Numbers



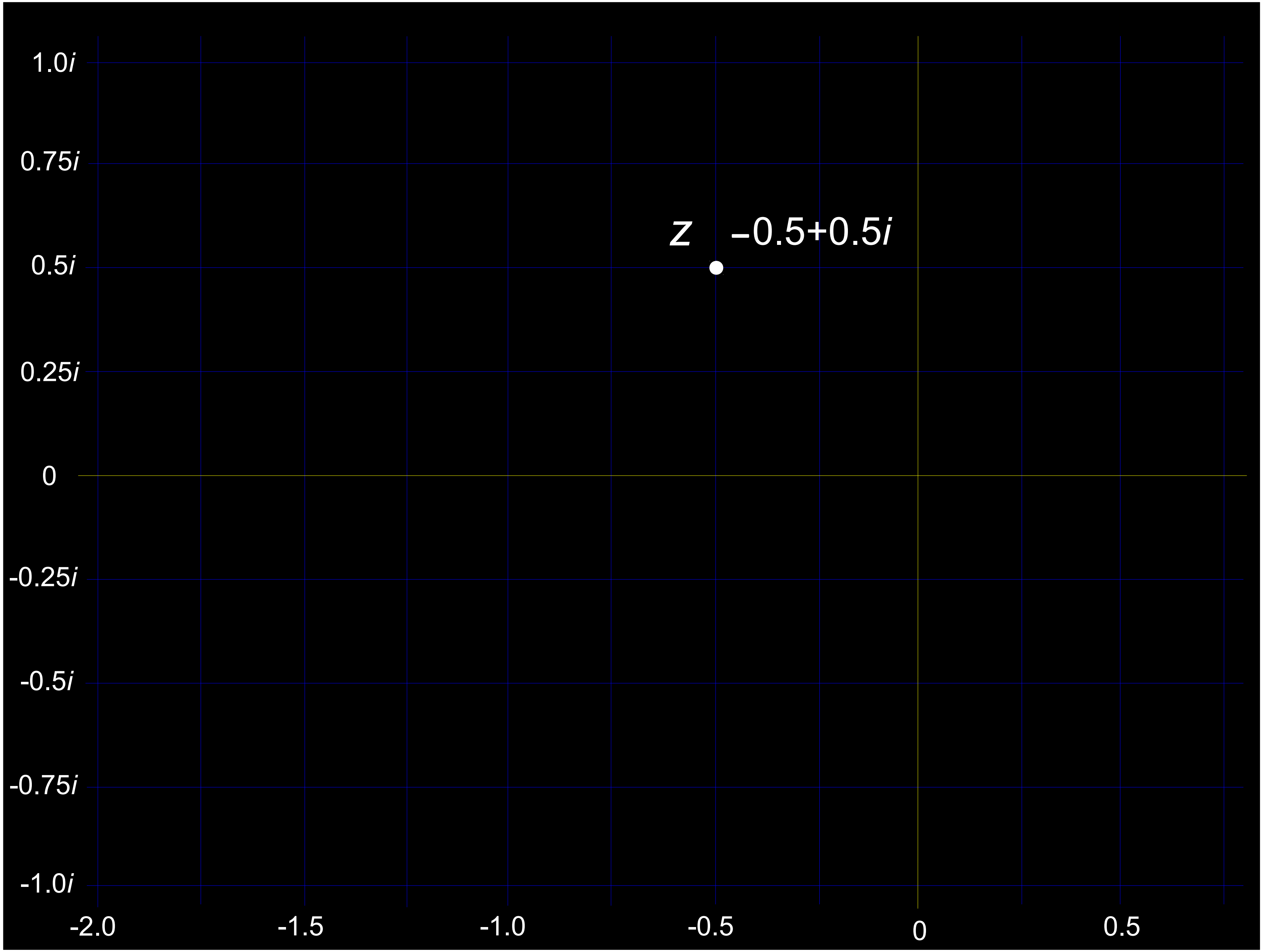
Complex Numbers



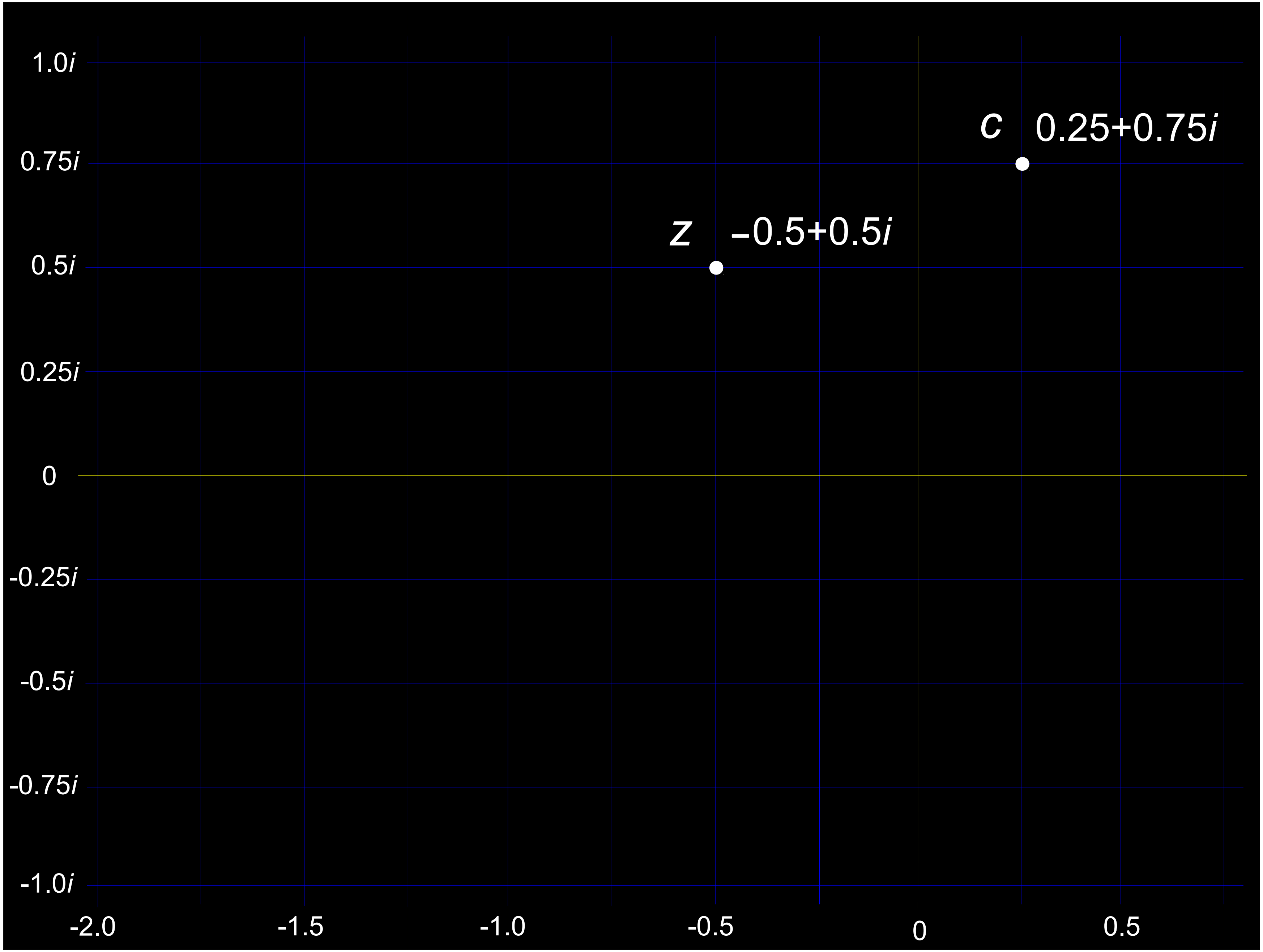
The Complex Plane



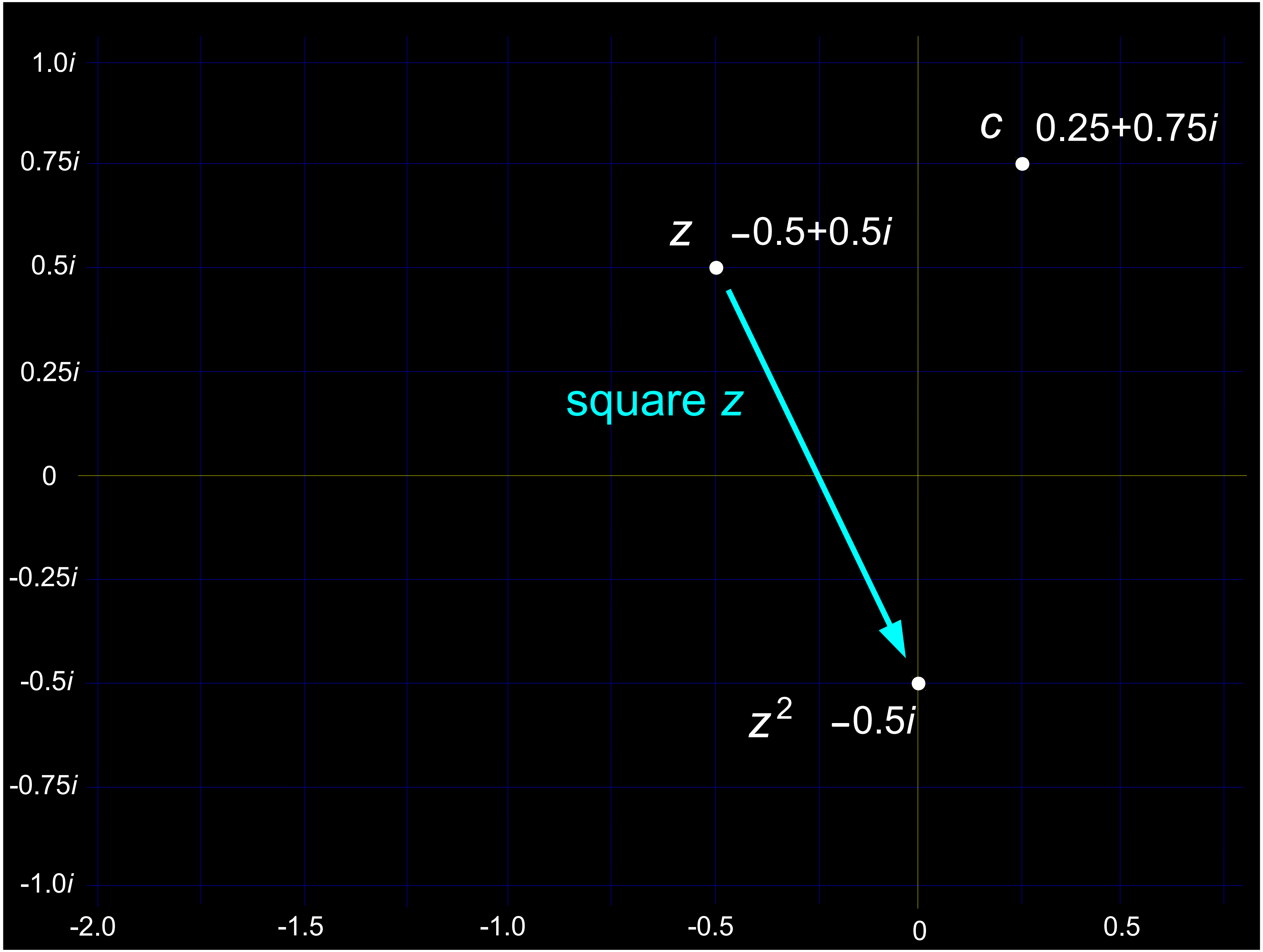
$z = -0.5+0.5i$



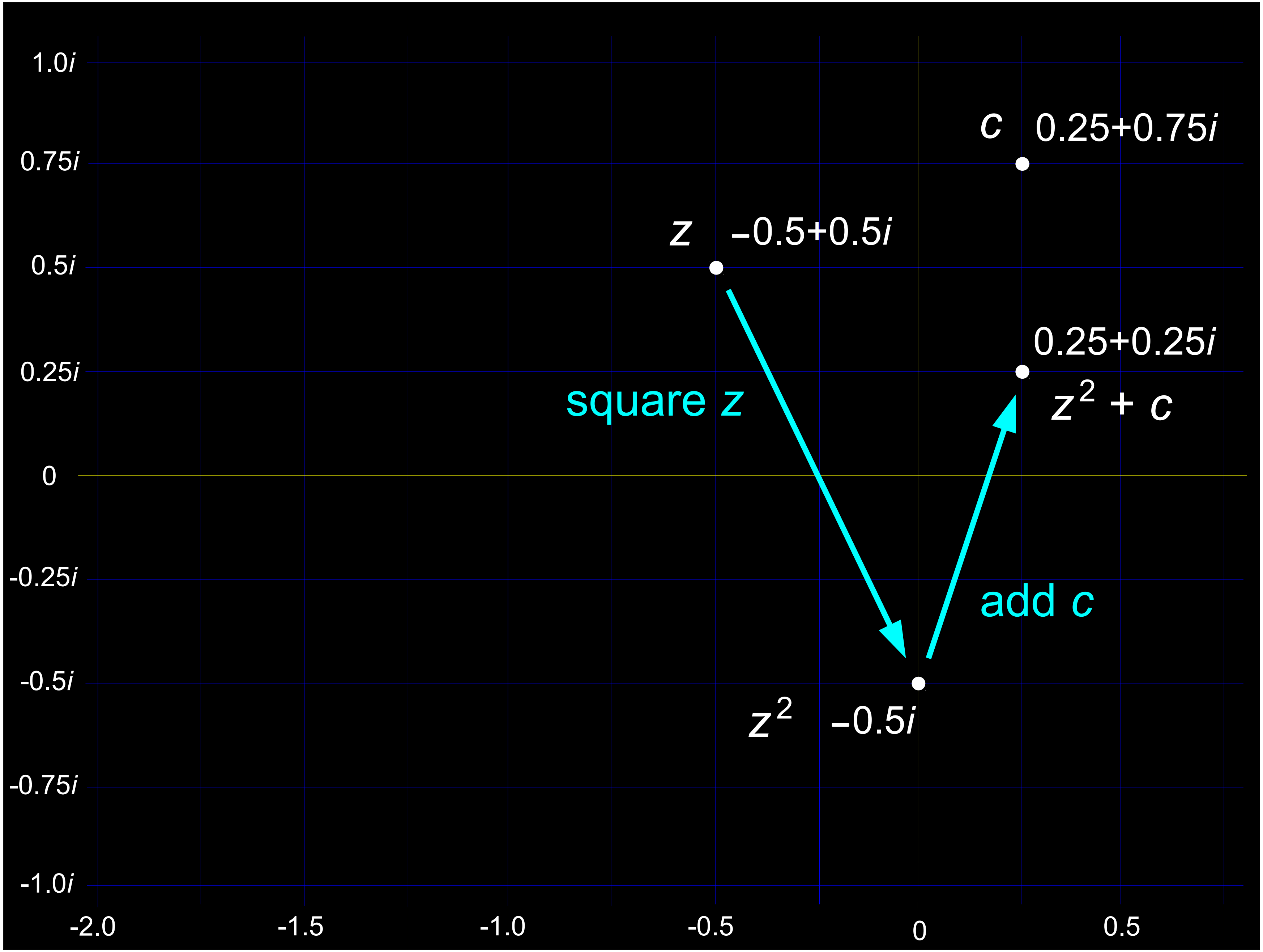
$z = -0.5+0.5i$ $c = 0.25+0.75i$



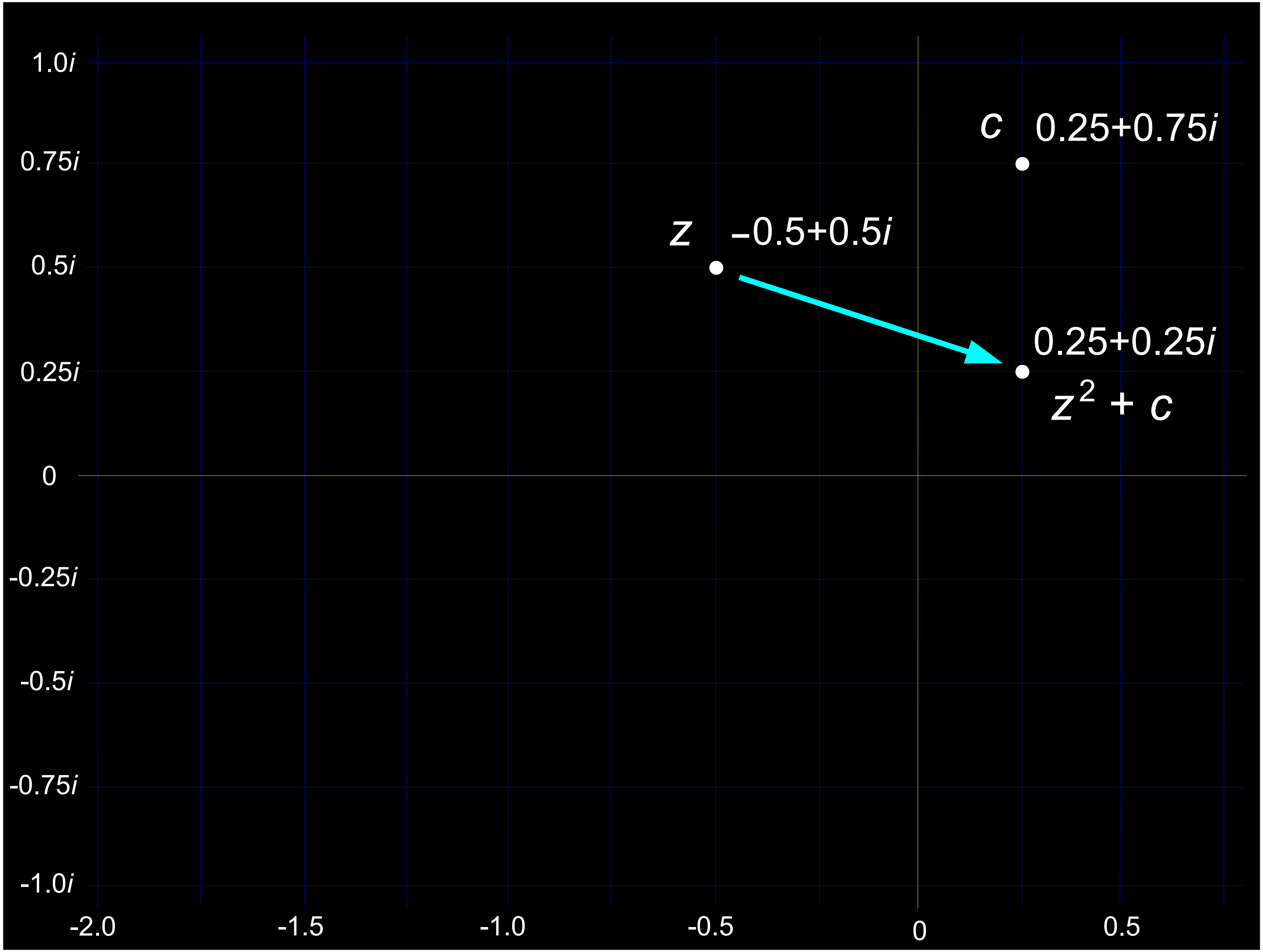
$z = -0.5+0.5i \qquad c = 0.25+0.75i$



$z = -0.5+0.5i \qquad c = 0.25+0.75i$



Notation: $z \longrightarrow z^2 + c$



Now, choose any complex number c

What happens when we repeatedly apply

$$z \rightarrow z^2 + c$$

starting with **zero** ?

Now, choose any complex number c

What happens when we repeatedly apply

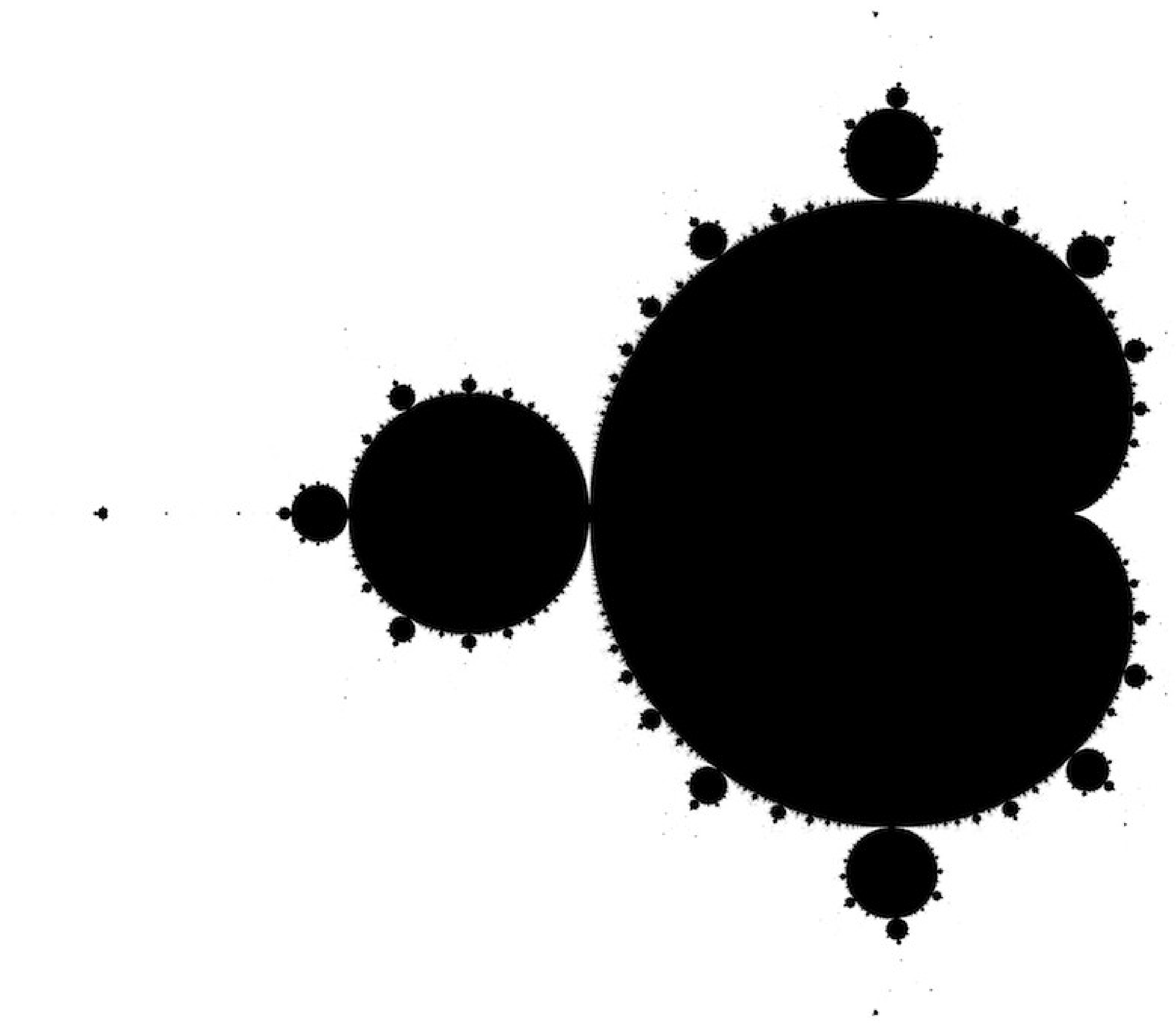
$$z \to z^2 + c$$

starting with **zero** ?

$$0 \;\to\; c \;\to\; c^2 + c \;\to\; (c^2 + c)^2 + c \;\to\; \ldots$$

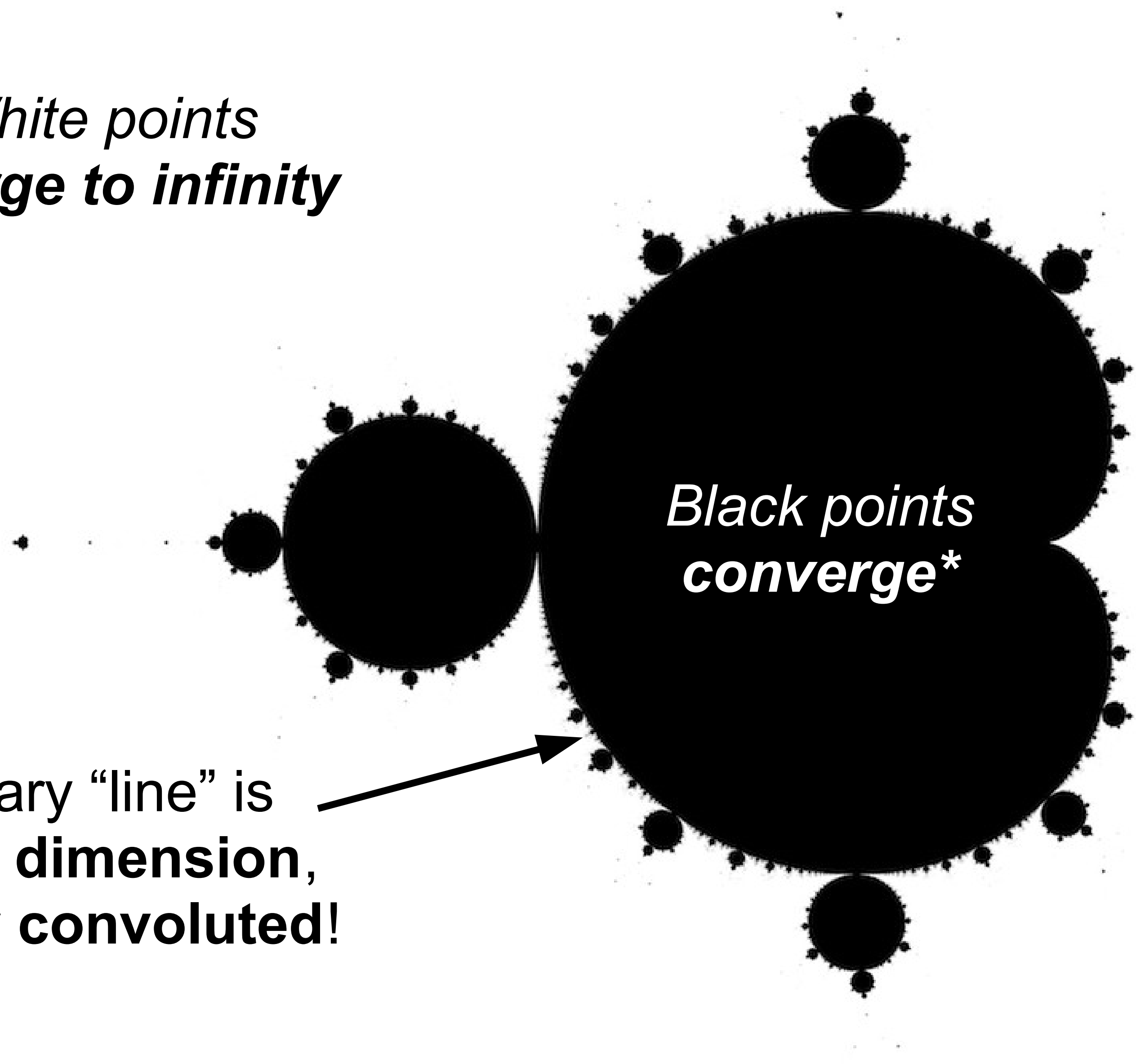
and so on ...

The Mandelbrot Set



The Mandelbrot Set

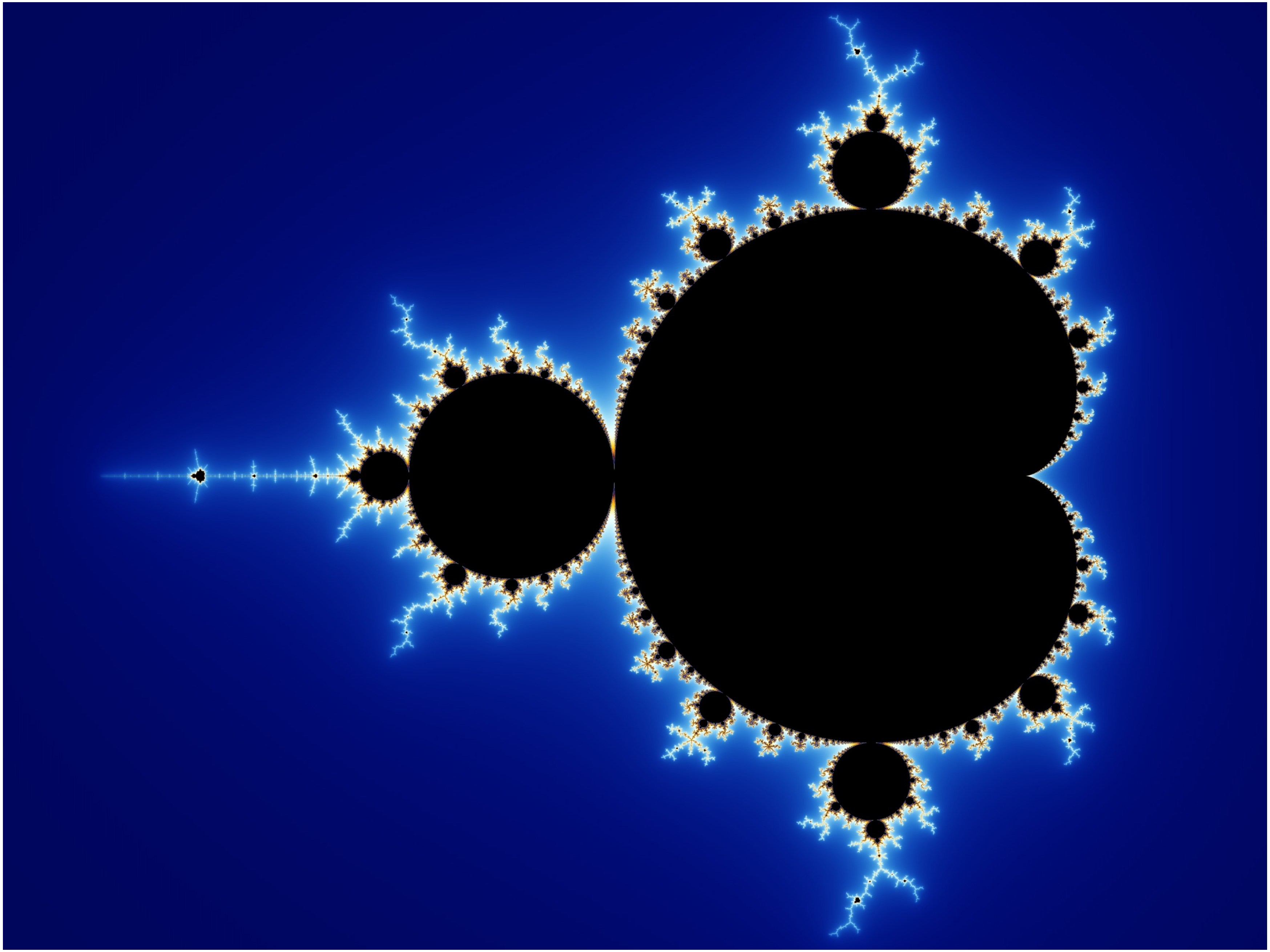
White points
diverge to infinity



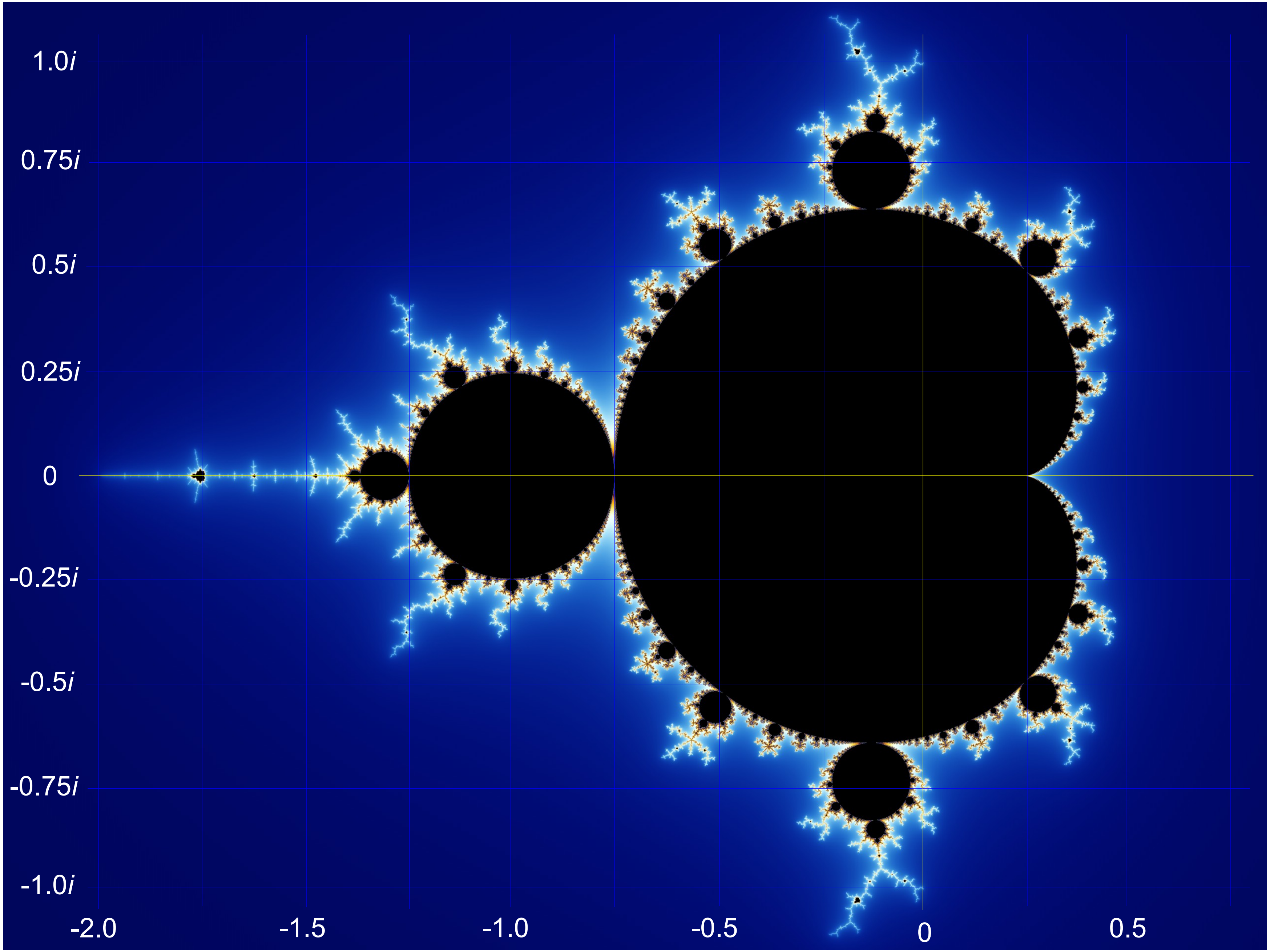
The boundary “line” is
of **fractional dimension**,
and **infinitely convoluted**!

* but not necessarily to zero, or even to a single, fixed point

The Mandelbrot Set



The Mandelbrot Set

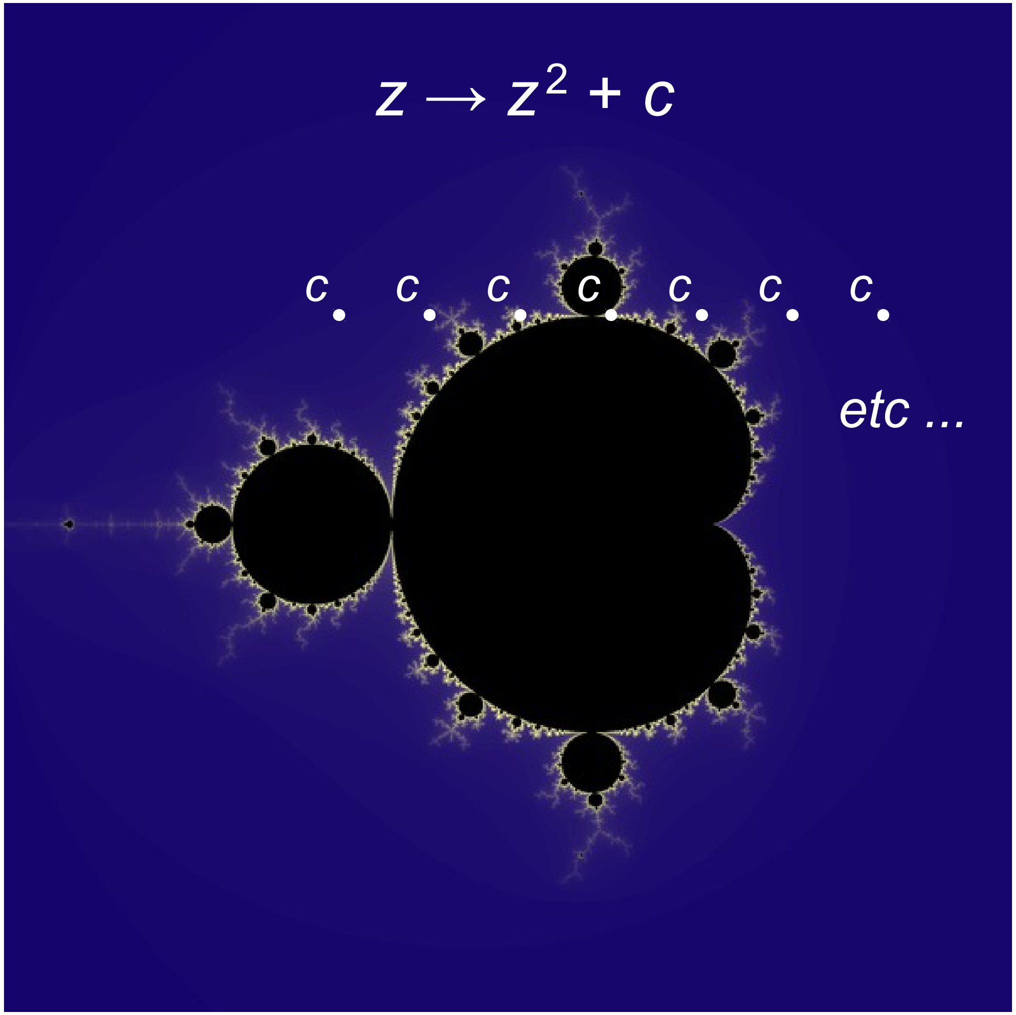


How to Color a Pixel

Let *c* be a complex number that corresponds to the pixel

- Initialize $z = 0$
- Repeatedly apply the update rule: $z \rightarrow z^2 + c$
- See how long it takes for the magnitude of *z* to exceed 2
  - If *z*'s magnitude never exceeds 2, color the pixel **black**
  - Otherwise, choose a color based on **how many steps** it took for *z*'s magnitude to exceed 2

How to Color a Pixel



We use different values for *c*, and always start the iteration at 0