Question:

Can we capture the concept of **primality** in a set of formal rules?

Why is 12 composite?

2 3 4 5 6 7 8 9 10 11 12

Why is 12 composite?

2 **3** **4** **5** **6** **7** **8** **9** **10** **11** **12**

2 divides 12

Why is 12 composite?

2 **3** 4 5 6 7 8 9 10 11 **12**

3 divides 12

Why is 12 composite?

2 3 **4** 5 6 7 8 9 10 11 **12**

4 divides 12

Why is 12 composite?

2 3 4 5 **6** 7 8 9 10 11 **12**

6 divides 12

Why is 11 prime?

2 **3** **4** **5** **6** **7** **8** **9** **10** **11** **12**

2 does not divide 11

Why is 11 prime?

2 **3** 4 5 6 7 8 9 10 **11** 12

3 does not divide 11

Why is 11 prime?

2 3 **4** 5 6 7 8 9 10 **11** 12

4 does not divide 11

Why is 11 prime?

2 3 4 **5** 6 7 8 9 10 **11** 12

5 does not divide 11

Why is 11 prime?

2 3 4 5 **6** 7 8 9 10 **11** 12

6 does not divide 11

Why is 11 prime?

2 3 4 5 6 **7** 8 9 10 **11** 12

7 does not divide 11

Why is 11 prime?

2 3 4 5 6 7 **8** 9 10 **11** 12

8 does not divide 11

Why is 11 prime?

2 3 4 5 6 7 8 **9** 10 **11** 12

9 does not divide 11

Why is 11 prime?

2 3 4 5 6 7 8 9 10 11 12

10 does not divide 11

Bigger numbers do not divide smaller numbers

12 does not divide 11

12 does not divide 6

3 does not divide 1

3 does not divide 2

15 does not divide 5

etc...

If x does not divide y,
then x does not divide $x+y$ either

3 does not divide 1, so it doesn't divide 4

3 does not divide 4, so it doesn't divide 7

3 does not divide 7, so it doesn't divide 10

3 does not divide 10, so it doesn't divide 13

etc...

n is divisor free up to x

25 is divisor free up to 4

2 3 4 5 6 7 8 9 10 11 12 13 . . . 24 **25**

49 is divisor free up to 6

2 3 4 5 6 7 8 9 10 11 12 13 . . . 48 **49**

11 is divisor free up to 10

2 3 4 5 6 7 8 9 10 11

If n is divisor free up to $n-1$, then n is prime

11 is divisor free up to 10, so 11 is prime

2	3	4	5	6	7	8	9	10	**11**

17 is divisor free up to 16, so 17 is prime

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**17**

The DND-system

Bigger numbers do not divide smaller numbers

Axiom Schema: xy**DND**x

where x and y are hyphen-strings

--DND-	2 does not divide 1
---DND-	3 does not divide 1
----DND---	4 does not divide 3
-----DND--	5 does not divide 2

etc...

If x does not divide y,
then x does not divide $x+y$ either

Rule: If x**DND**y is a theorem, so is x**DND**xy

---DND-	*(axiom)*	3 does not divide 1
---DND----		3 does not divide 4
---DND-------		3 does not divide 7
---DND----------		3 does not divide 10
etc...		
----DND--	*(axiom)*	4 does not divide 2
----DND------		4 does not divide 6
etc...		

If 2 does not divide n,
then n is divisor free up to 2

Rule: If **--DND**n is a theorem, so is n**DF--**

--DND-	*(axiom)*	2 does not divide 1
--DND---		2 does not divide 3
---DF--		3 is divisor free up to 2
--DND-----		2 does not divide 5
-----DF--		5 is divisor free up to 2
--DND---------		2 does not divide 9
---------DF--		9 is divisor free up to 2

If n is divisor free up to x,
and x+1 does not divide n,
then n is divisor free up to x+1

Rule: If n**DF**x and x**–DND**n are both theorems, so is n**DF**x**–**

------DF--	5 is divisor free up to 2
---DND------	3 does not divide 5
------DF---	5 is divisor free up to 3
----DND------	4 does not divide 5
------DF----	5 is divisor free up to 4

If n is divisor free up to $n-1$, then n is prime

Rule: If z**-DF**z is a theorem, so is **P**z**-**

---DF--	3 is divisor free up to 2
P---	3 is prime
-----DF----	5 is divisor free up to 4
P-----	5 is prime

Axiom: **P--** 2 is prime

Answer:

Yes, the prime numbers can be **mechanically generated** by a set of formal rules!