

## ***Practice Exercises – Propositional Calculus and TNT***

**Due by class time Friday, December 9**

1. Derive the following theorem of Propositional Calculus (hint: the Fantasy Rule will come in handy). Write out your derivation in the same style as shown on pages 189-190 of *GEB*.

$$\langle \sim \langle P \wedge Q \rangle \supset \langle \sim P \vee \sim Q \rangle \rangle$$

2. Tommy Flanagan was telling you what he ate yesterday afternoon. He tells you, “I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea.” Of course you know that Tommy is the world's worst liar, and everything he says is false. What did Tommy eat or drink?

Using the symbols  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$  (defined below), write down each of Tommy's statements as negations, and then use the rules of the Propositional Calculus to deduce what Tommy actually ate or drank. For example, the negation of his first statement would be encoded as  $\sim \langle P \vee R \rangle$ .

$P$  = ate popcorn

$Q$  = ate cucumber sandwiches

$R$  = ate raisins

$S$  = drank soda

$T$  = drank tea

3. Translate the following English sentences into equivalent TNT formulas:
  - (a) “Every natural number is equal to itself multiplied by 1”
  - (b) “Multiplication of natural numbers is commutative”
  - (c) “If  $b$  is not zero, then there exists some number  $a$  whose successor is  $b$ ”
  - (d) “If the product of any two numbers is zero, then one of the numbers must be zero”
  - (e) “ $b$  is greater than 2”

4. Translate the following TNT formulas into clear and concise English sentences (which may or may not be true) describing properties of natural numbers:

(a)  $\forall c: \forall d: \langle (c+d)=0 \supset \langle c=0 \wedge d=0 \rangle \rangle$

(b)  $\sim \exists b: (b \cdot b)=SSSS0$

(c)  $\sim \forall b: \exists a: Sa=b$

5. Is the TNT string below well-formed? That is the question. Explain your answer clearly and concisely.

$$\langle (SS0 \cdot b) \vee \sim(SS0 \cdot b) \rangle$$

6. The statement “different numbers have different successors” can be translated into TNT notation as:

$$\forall a: \forall b: \langle \sim a=b \supset \sim Sa=Sb \rangle$$

Derive this string as a theorem of TNT, showing each step of the derivation along with the rule used. Hint: start with the Fantasy Rule.