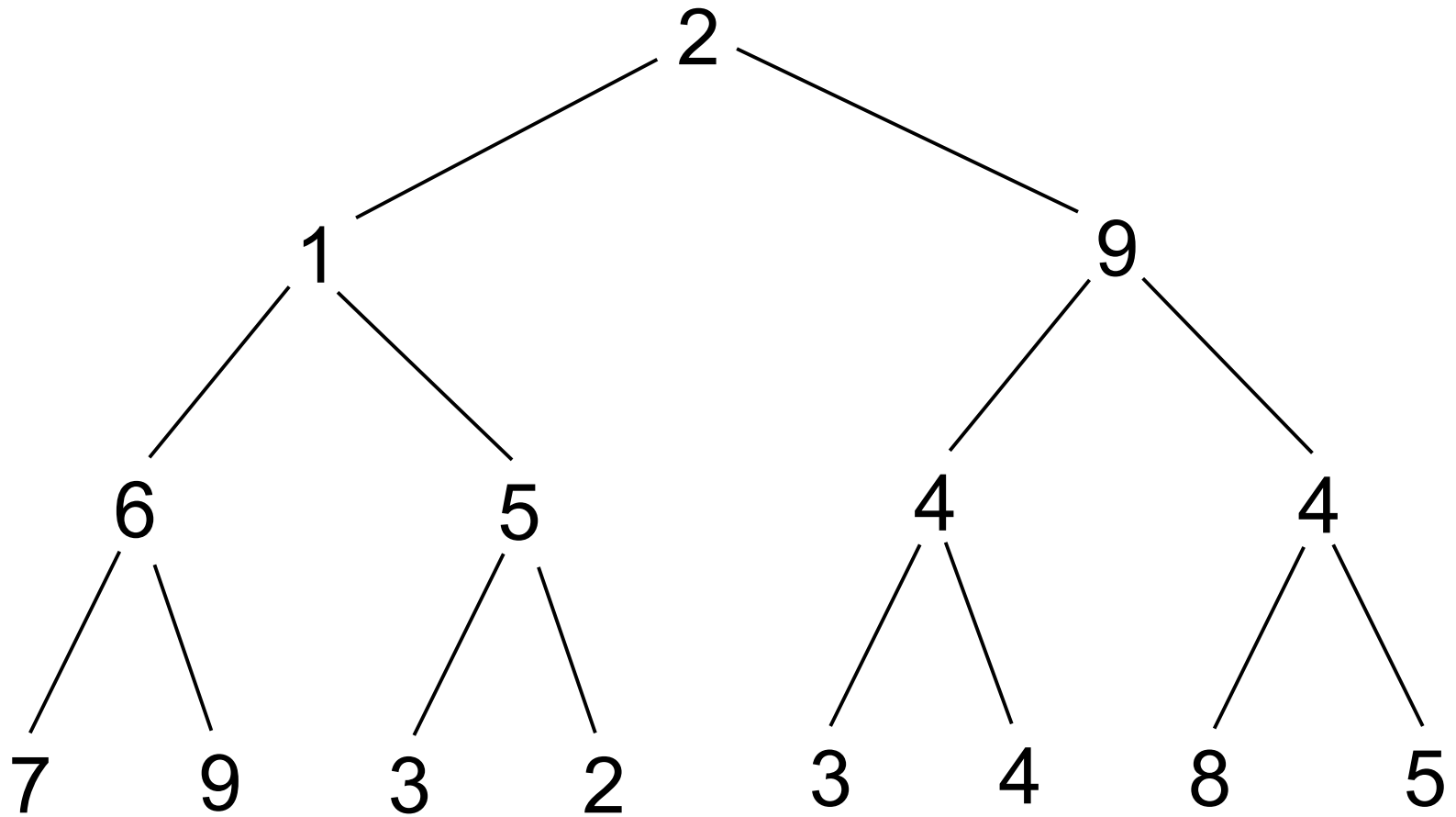
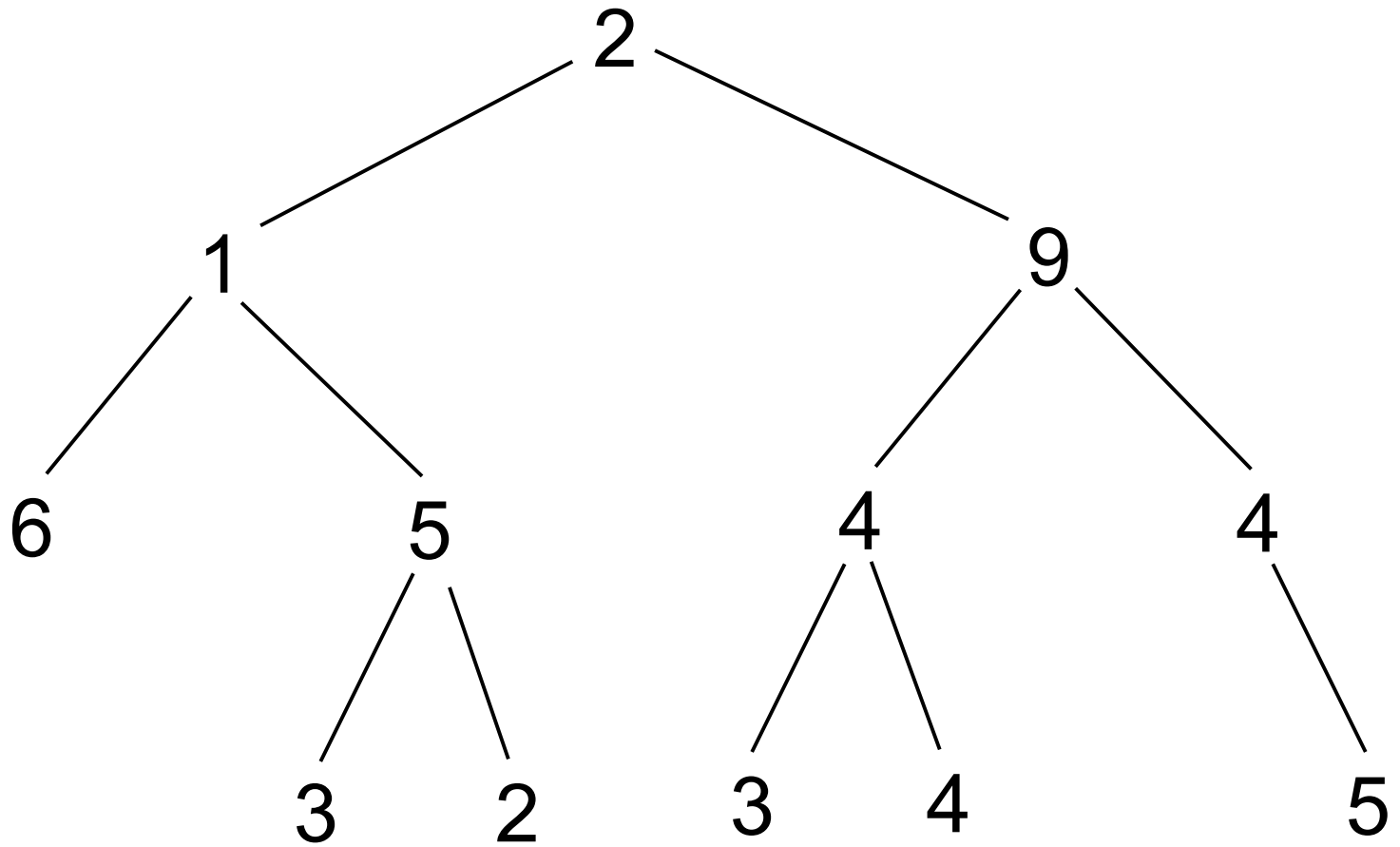


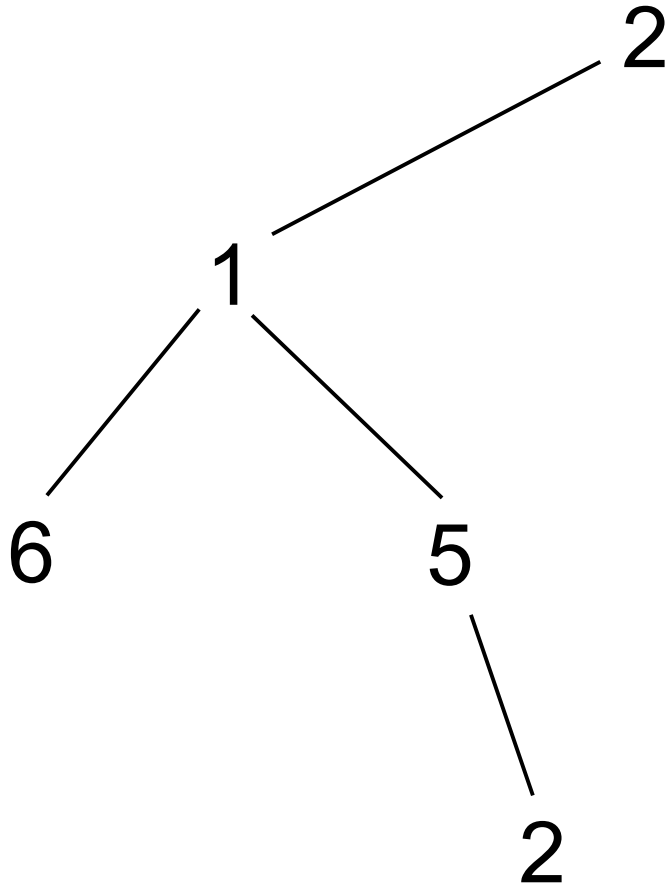
Binary Trees



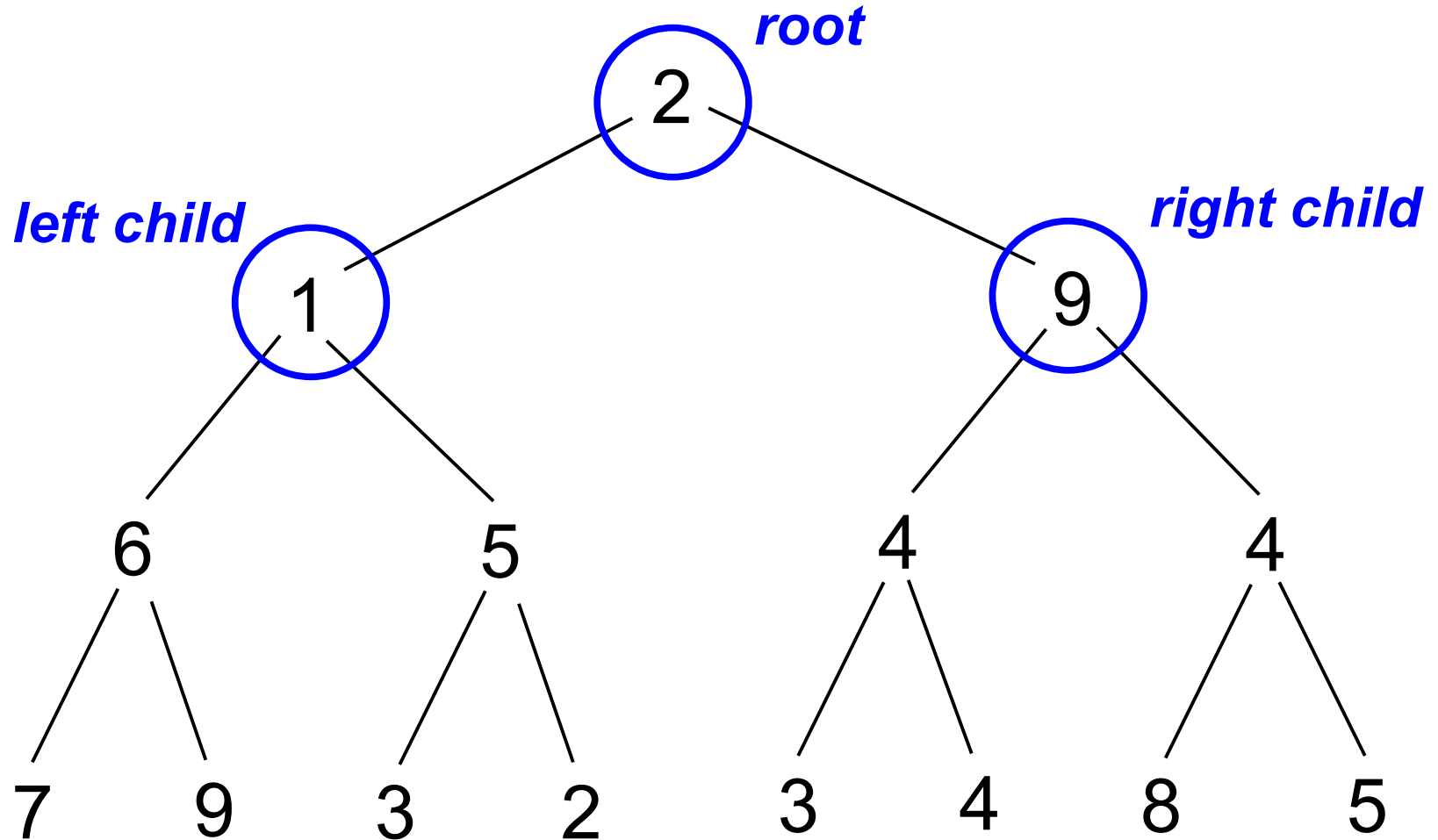
Binary Trees



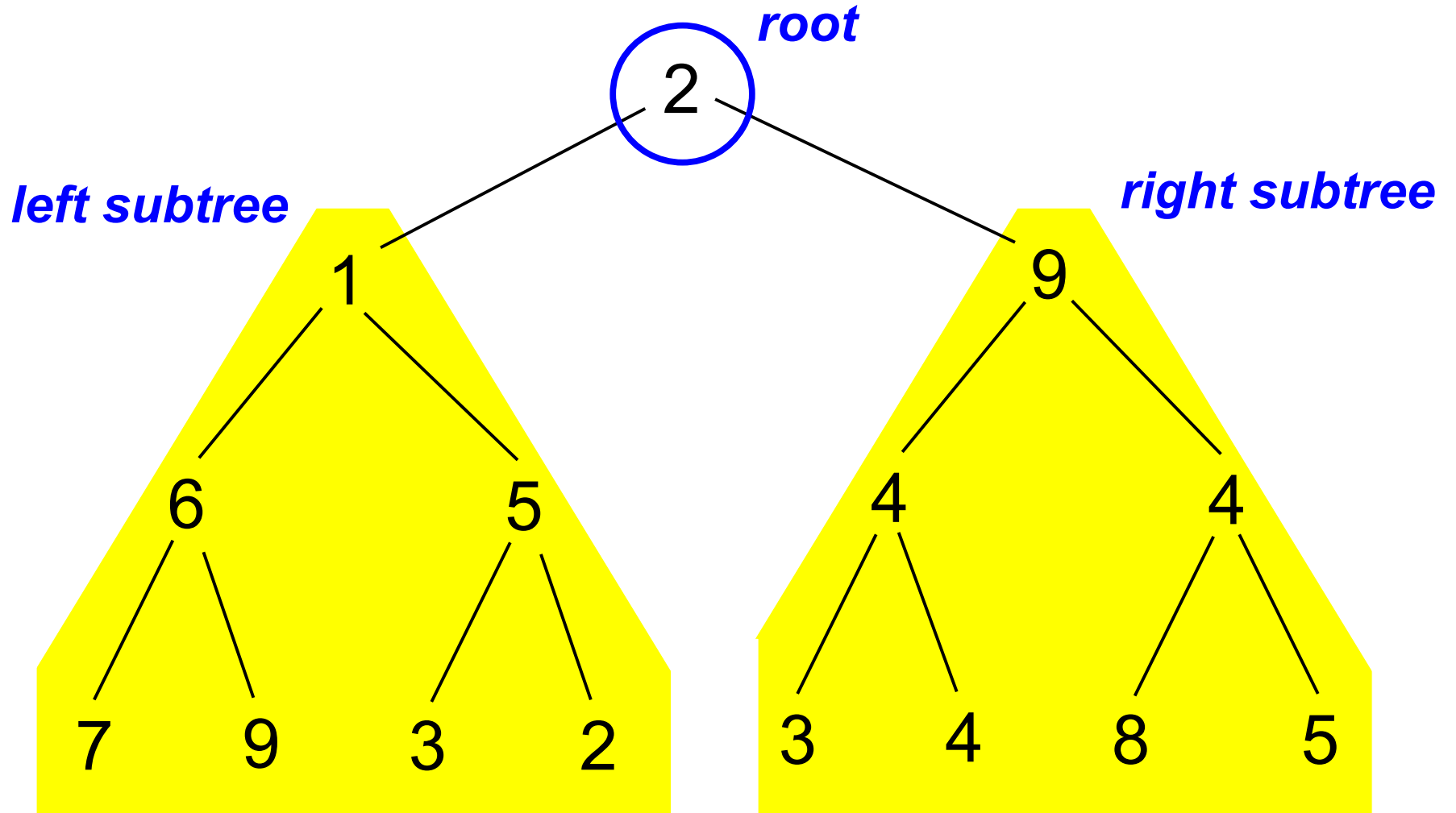
Binary Trees



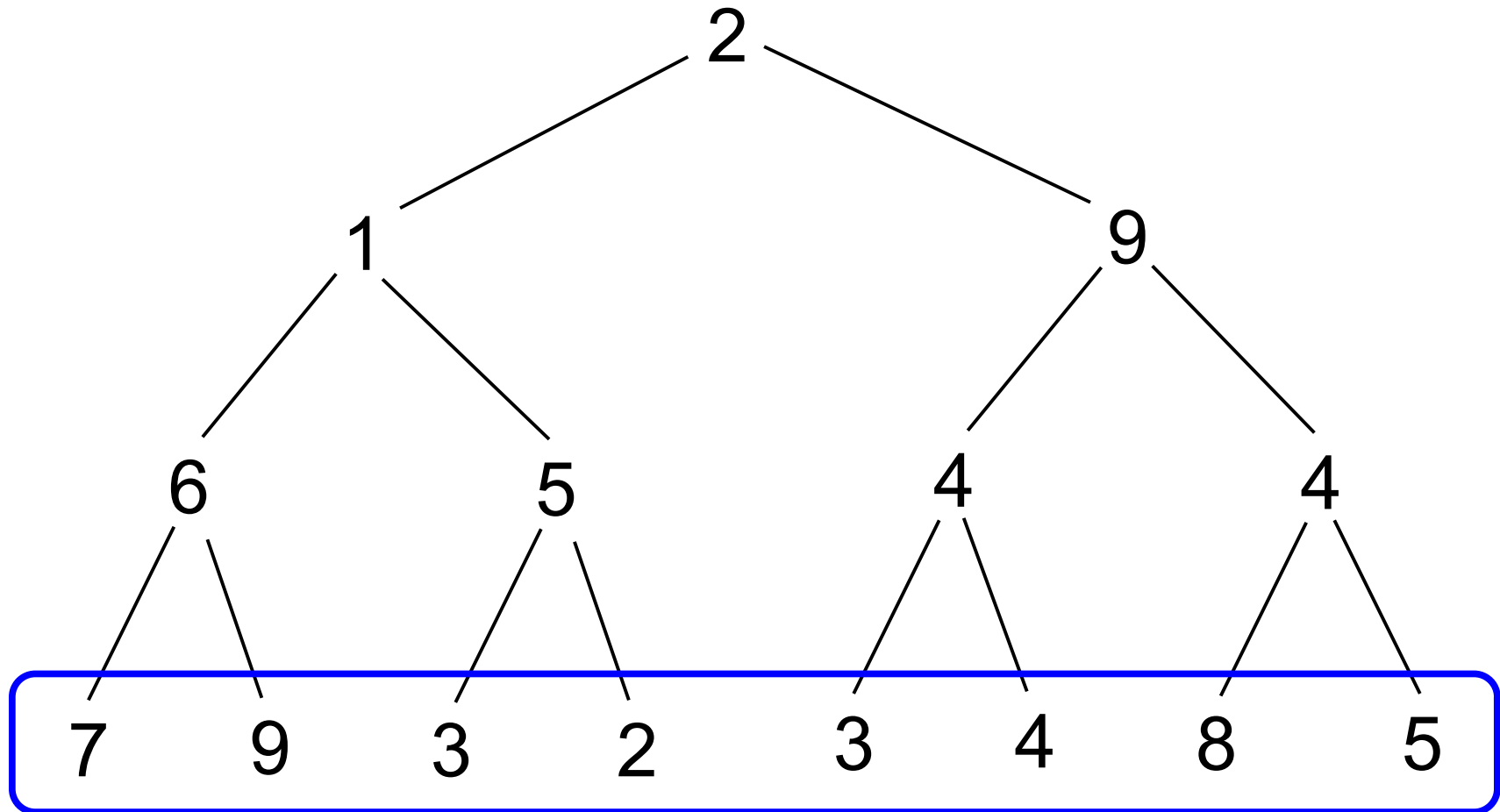
Terminology



Terminology

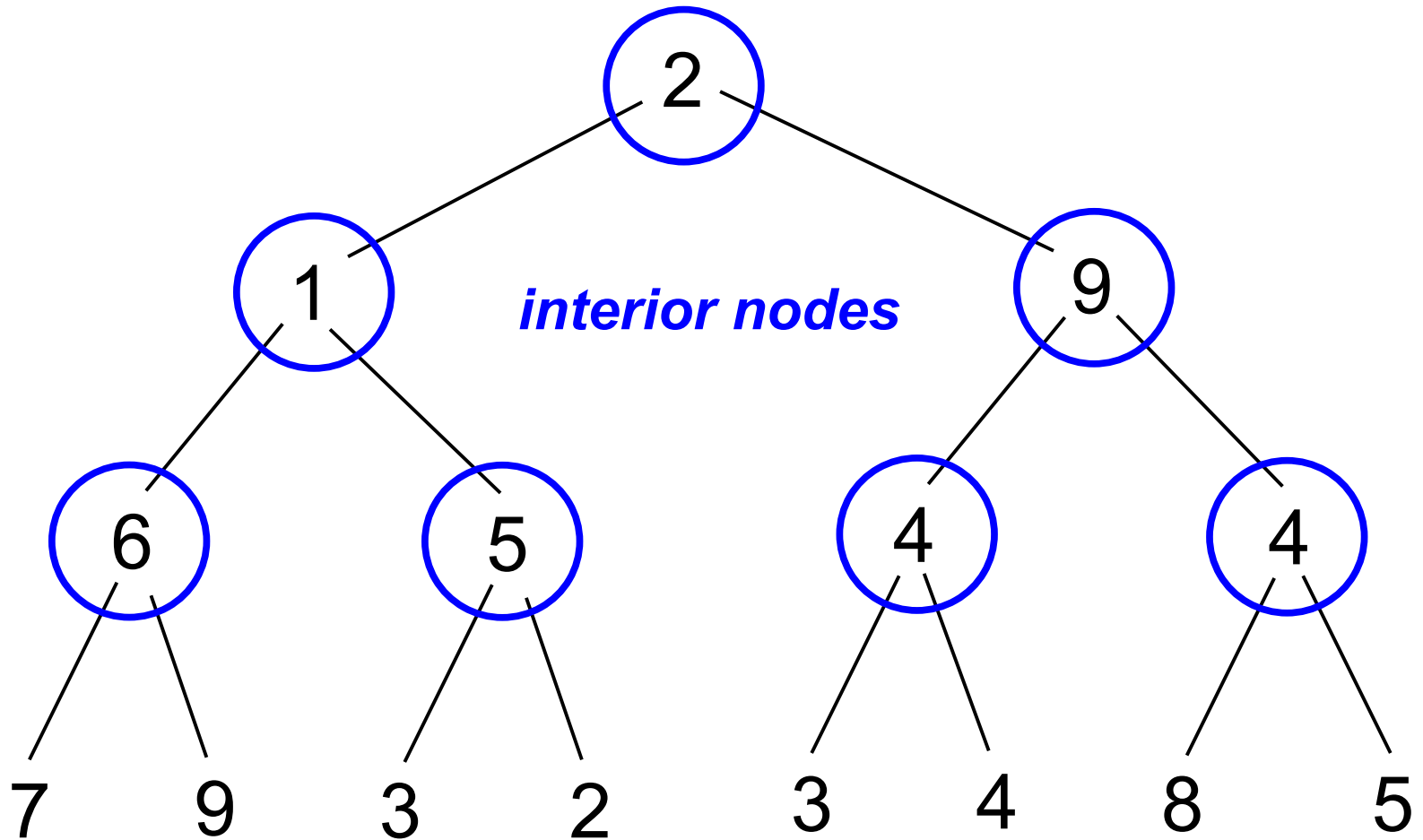


Terminology

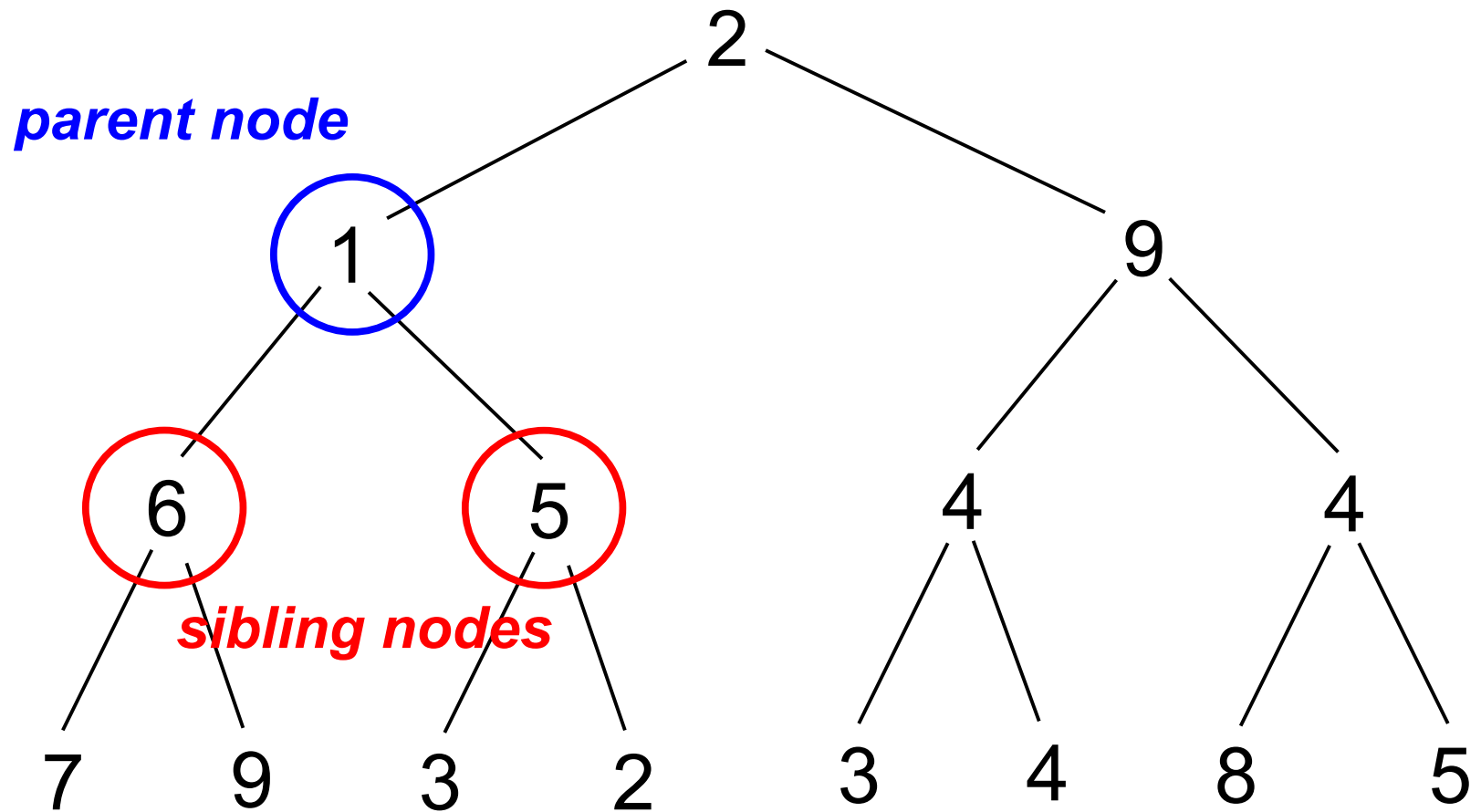


leaf nodes

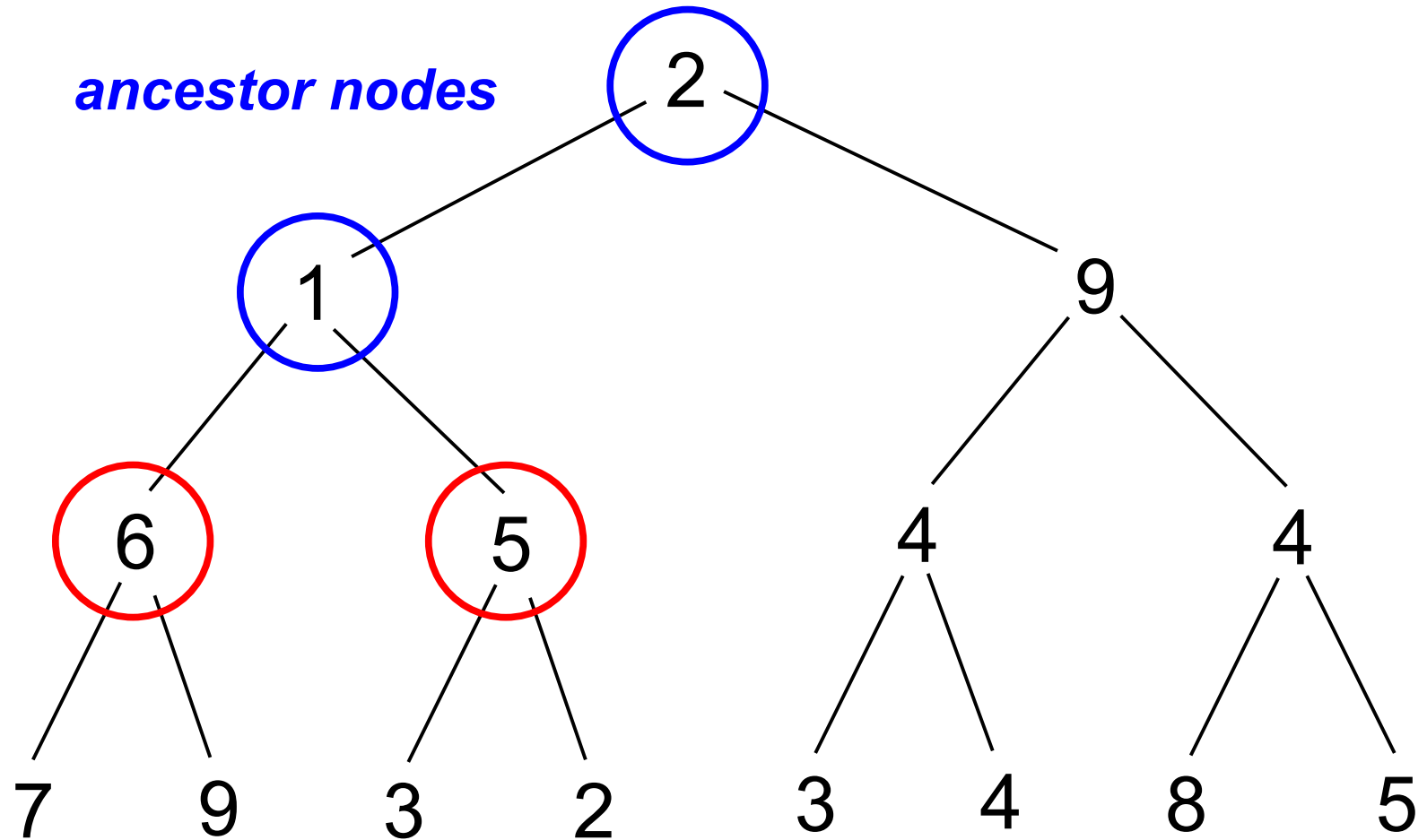
Terminology



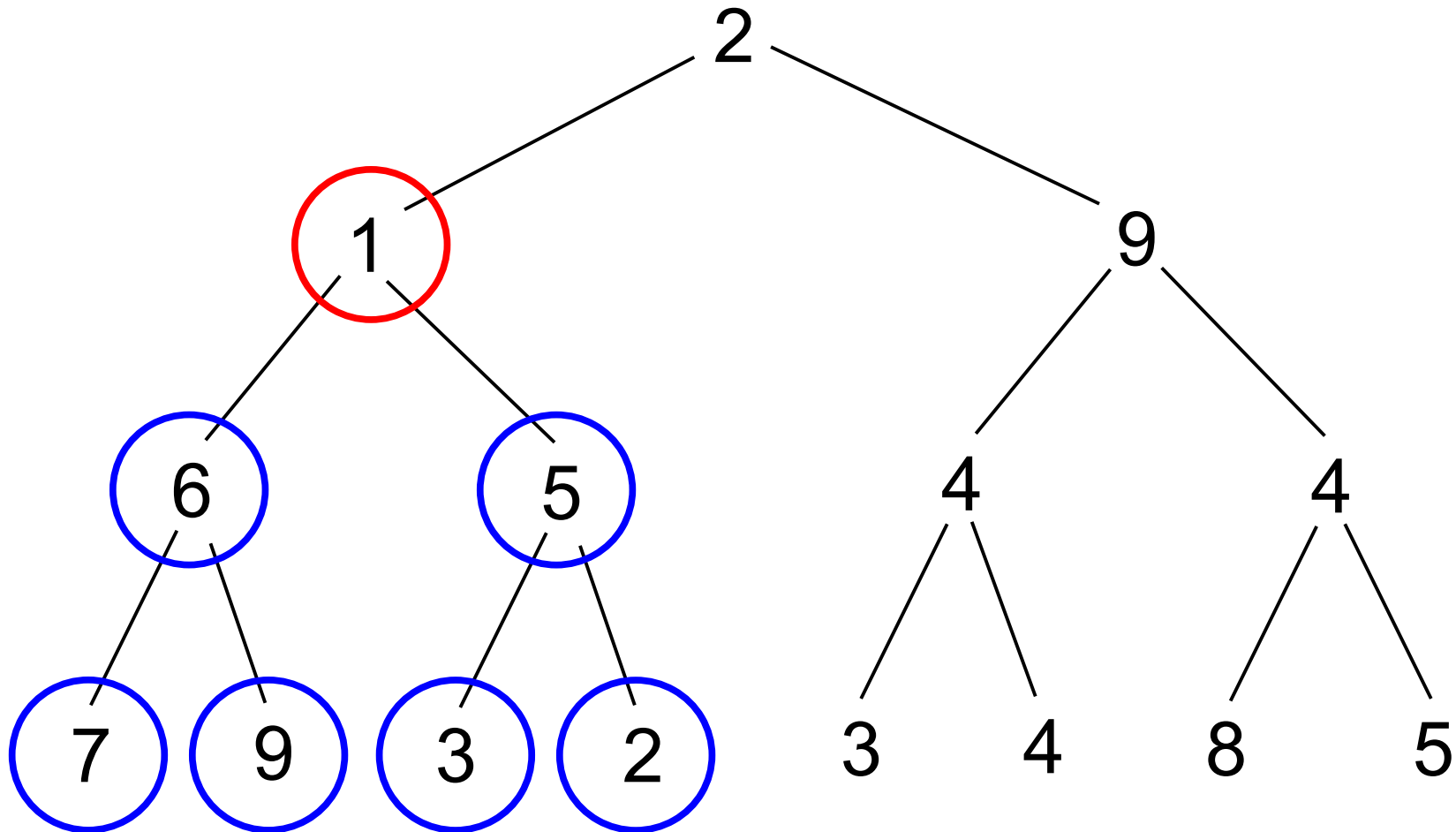
Terminology



Terminology

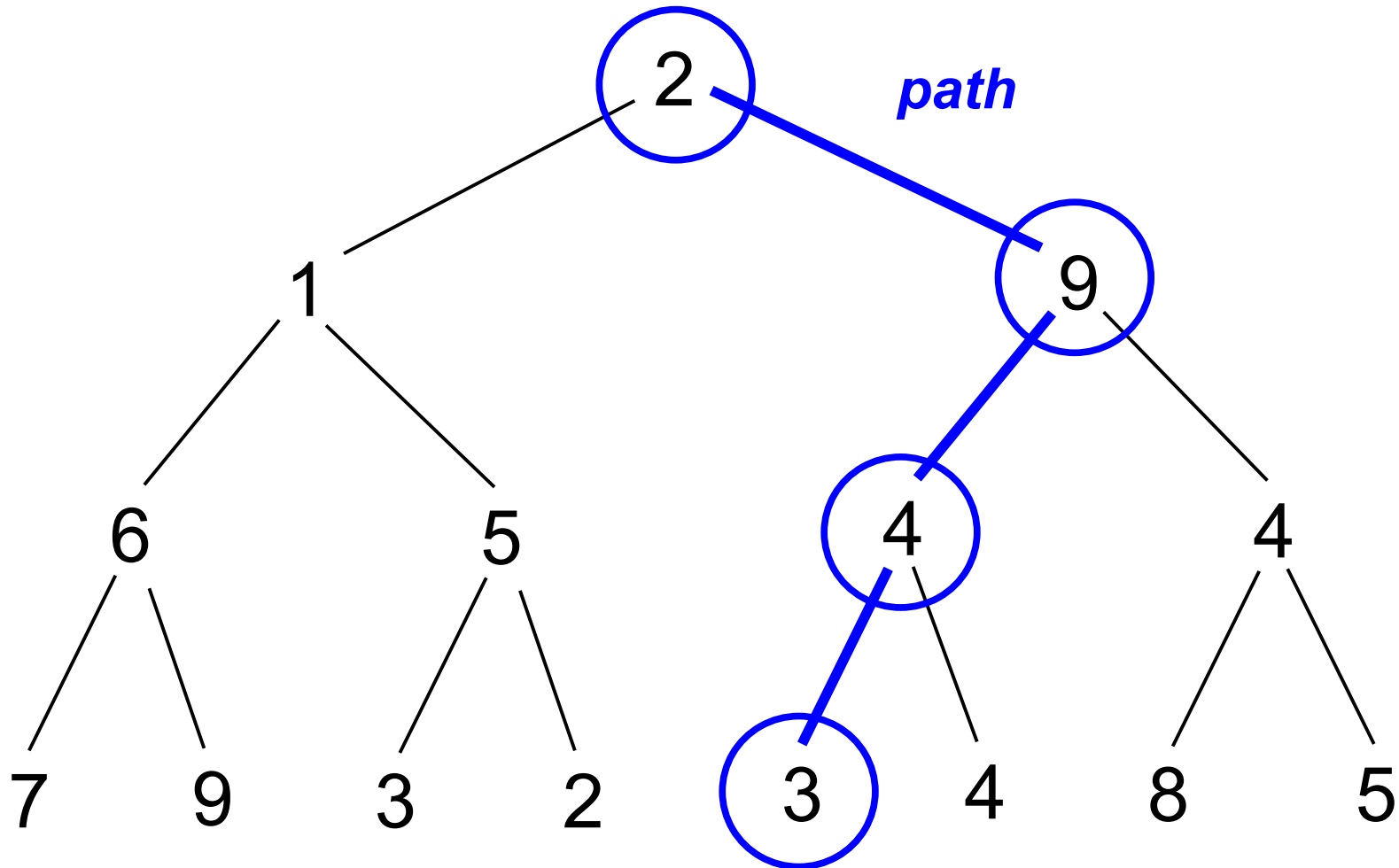


Terminology



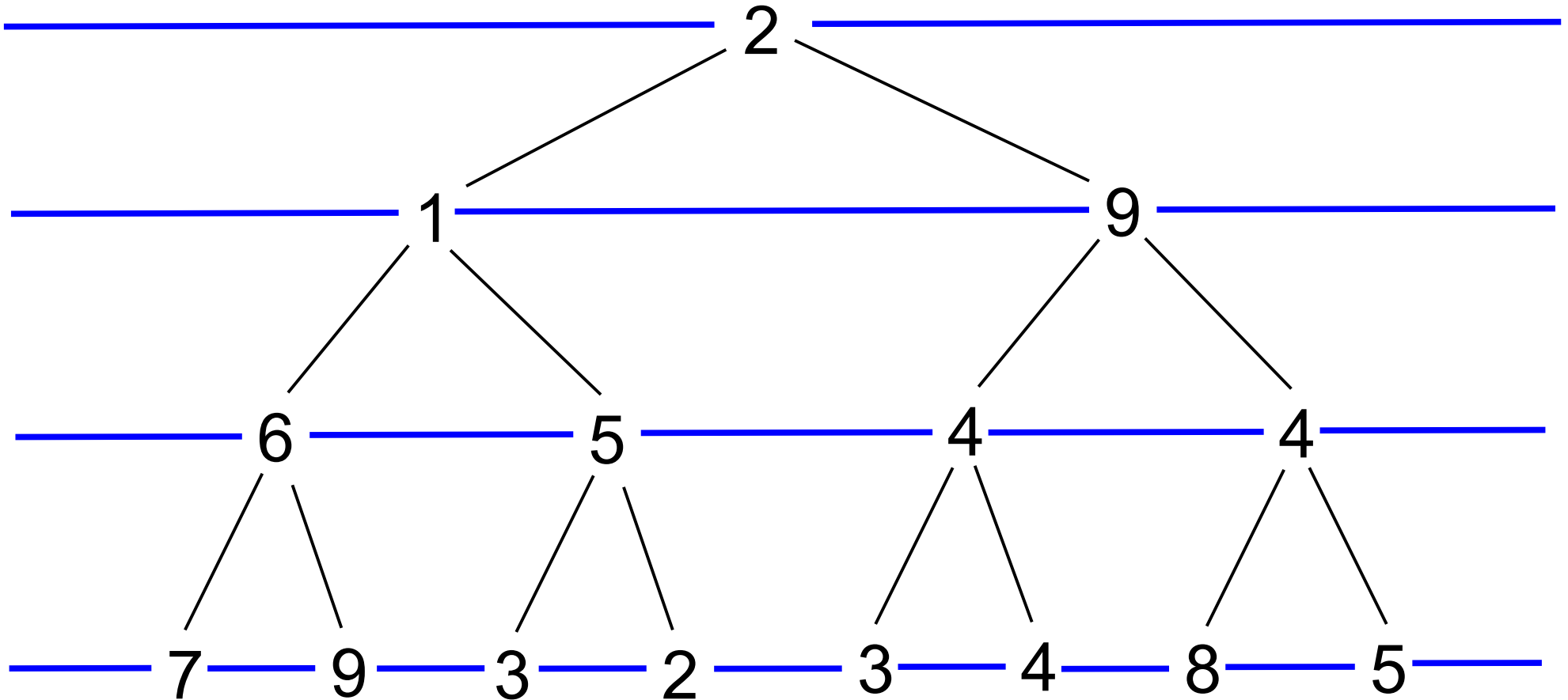
descendant nodes

Terminology



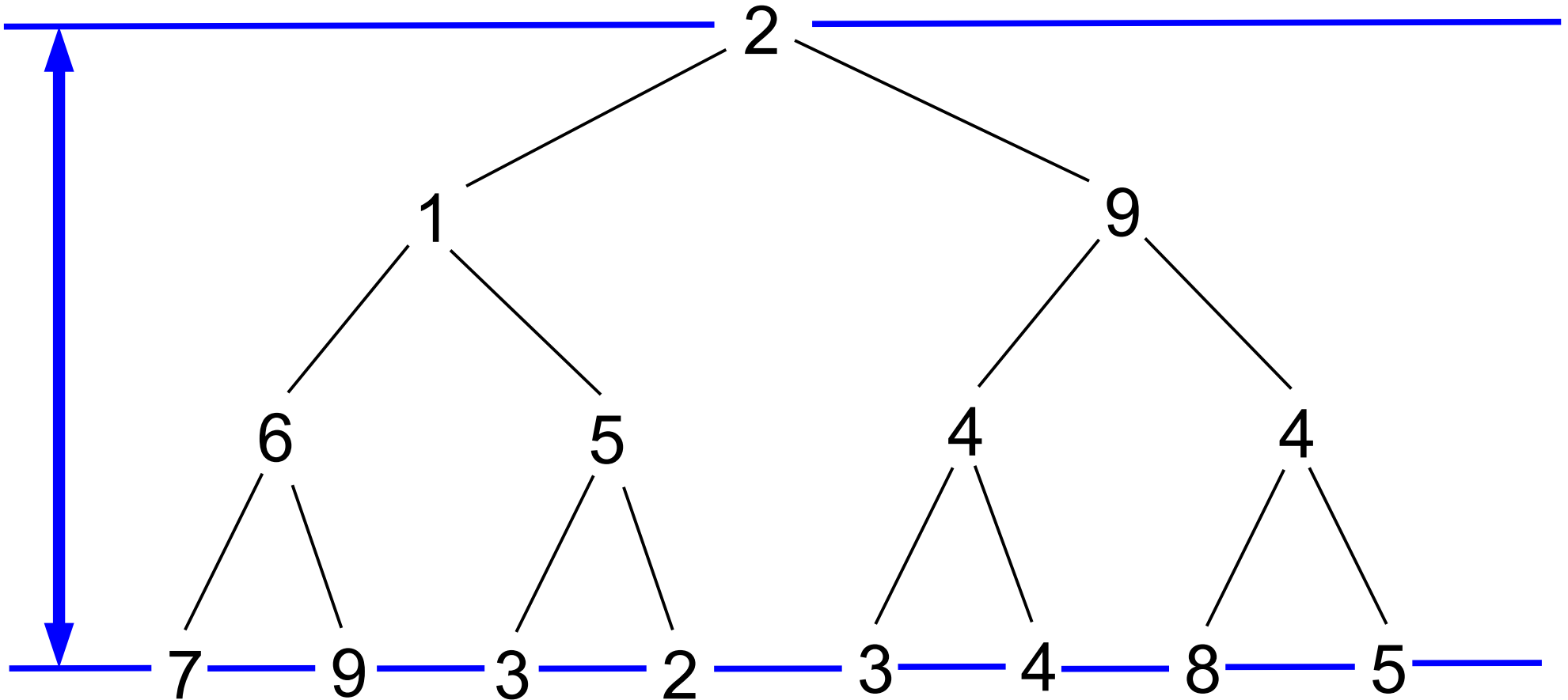
Terminology

levels



Terminology

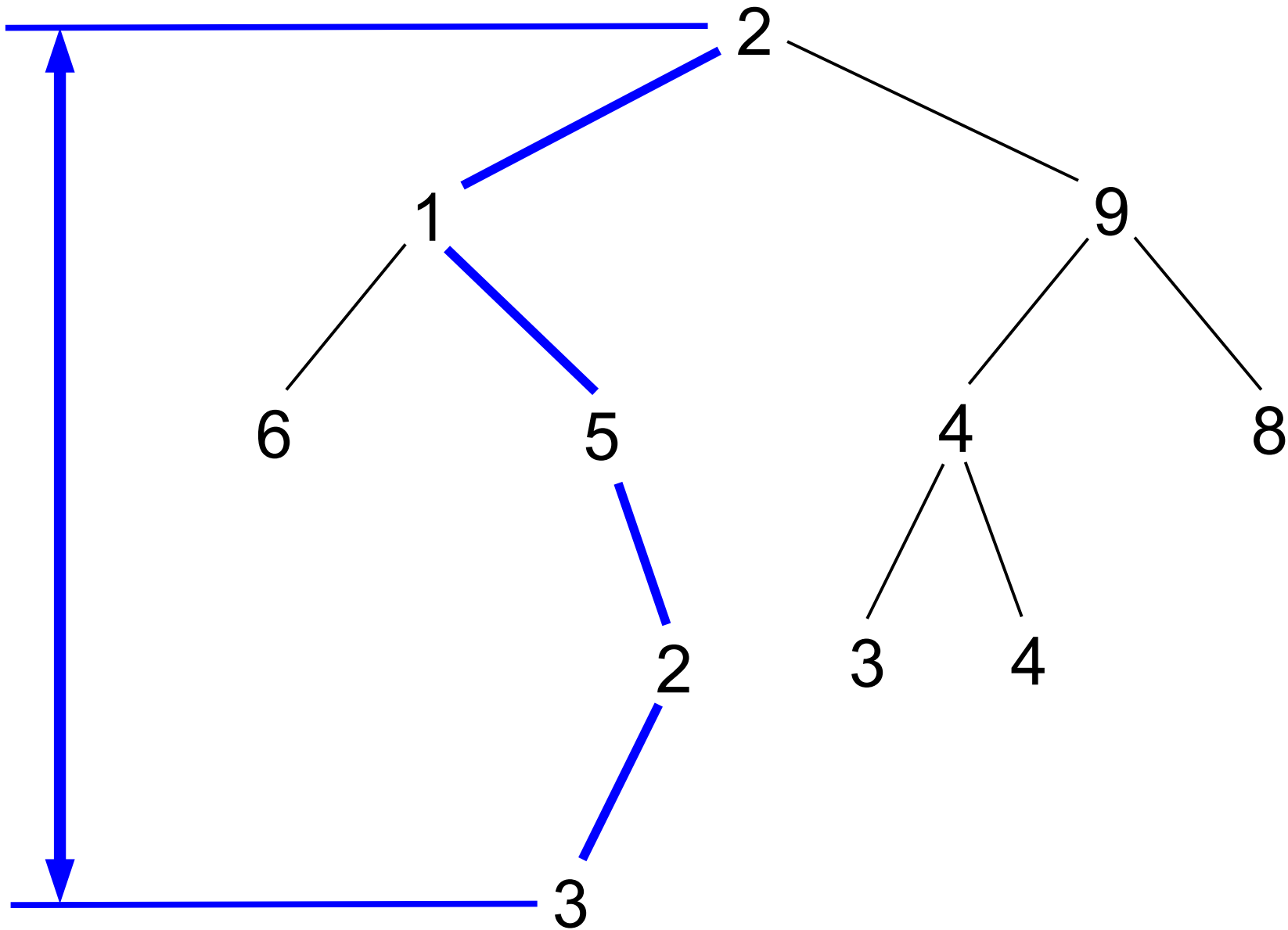
height = 3



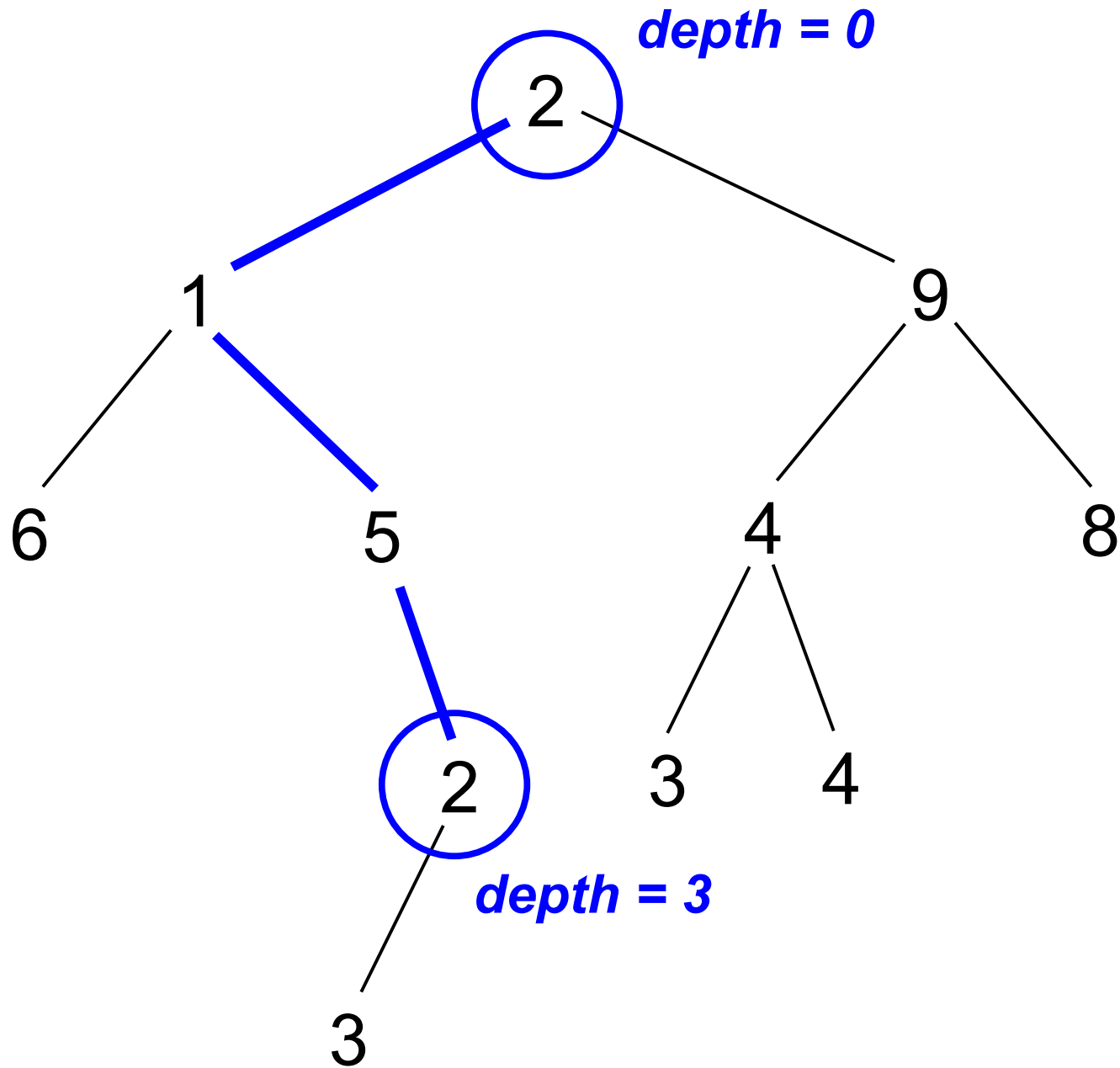
Note: this usage differs from the book's usage

Terminology

height = 4

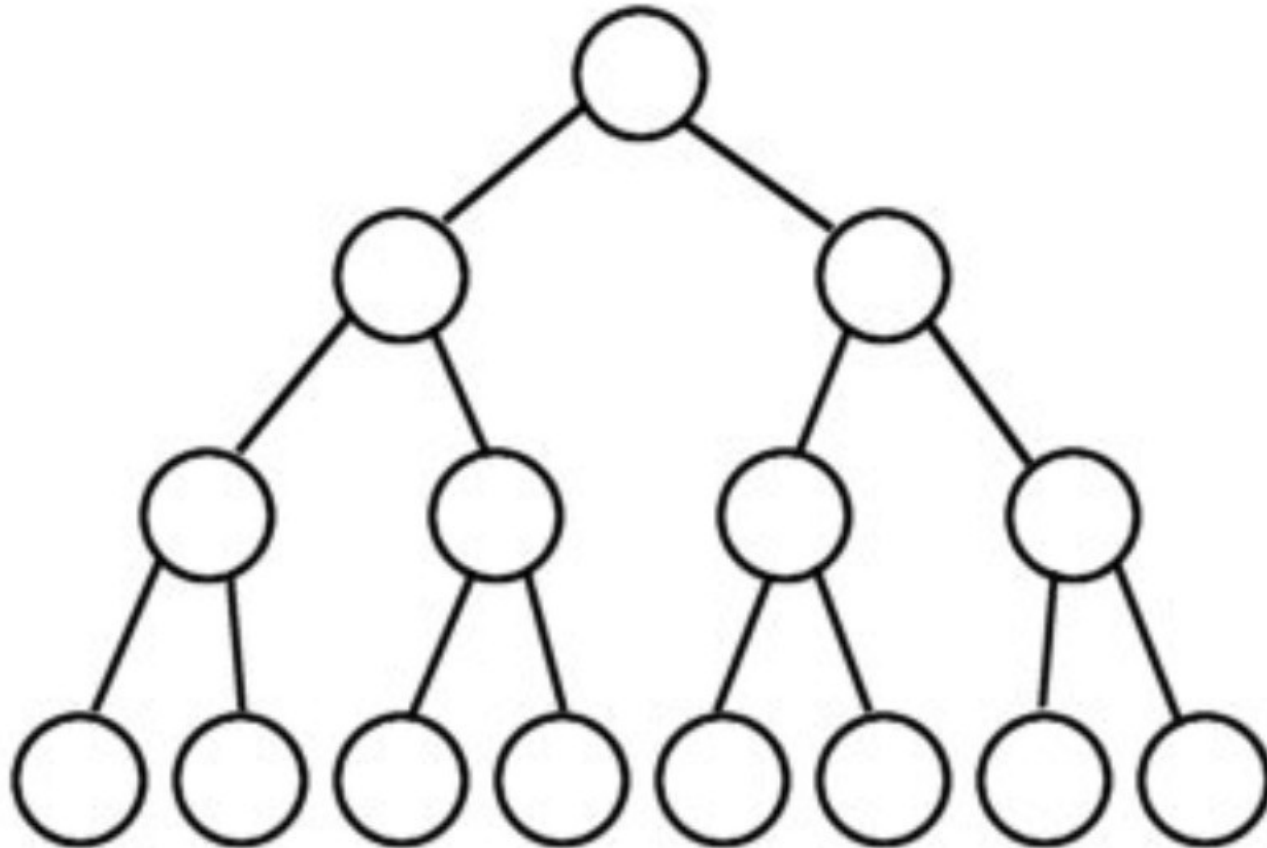


Terminology



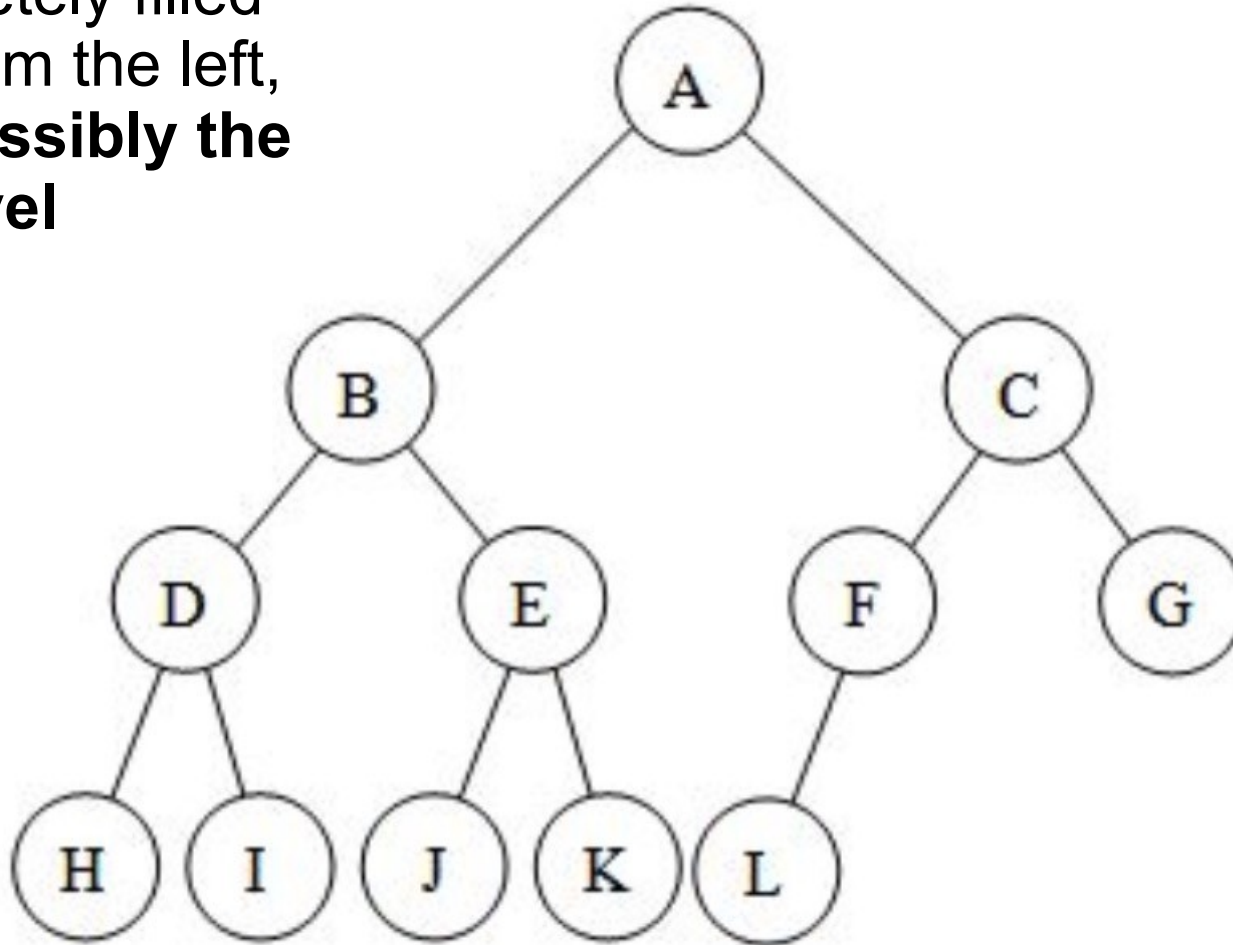
Full Binary Tree

All nodes have
either 2 or 0 children

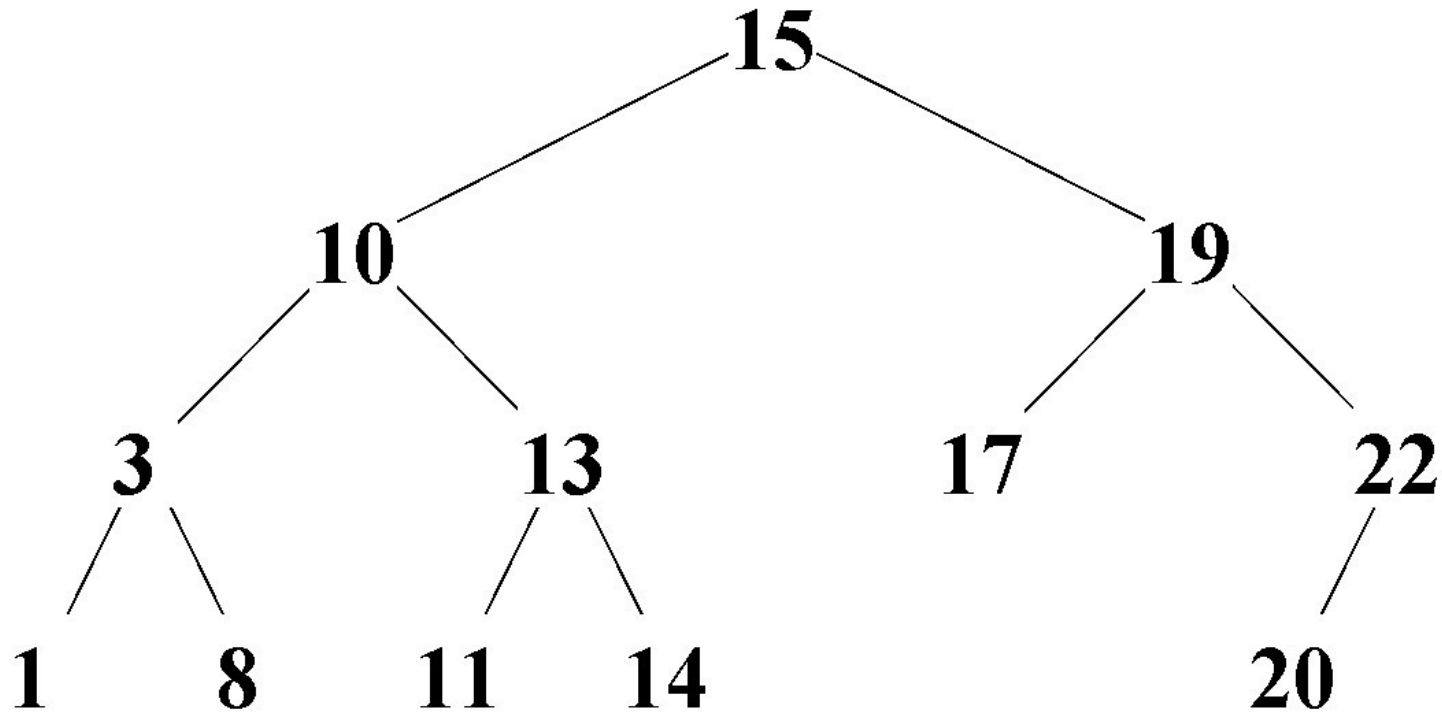


Complete Binary Tree

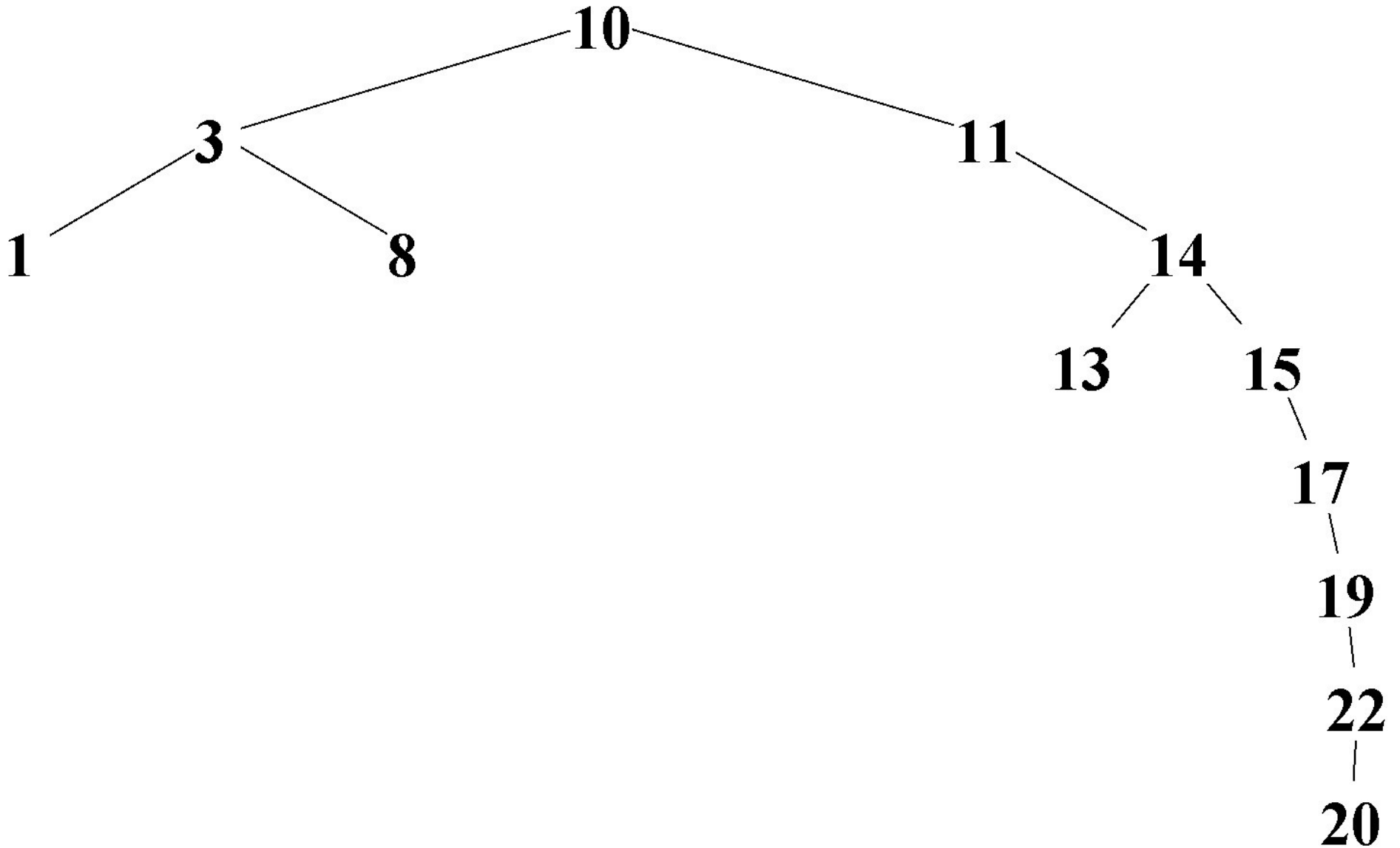
All levels of the tree are completely filled starting from the left, **except possibly the lowest level**



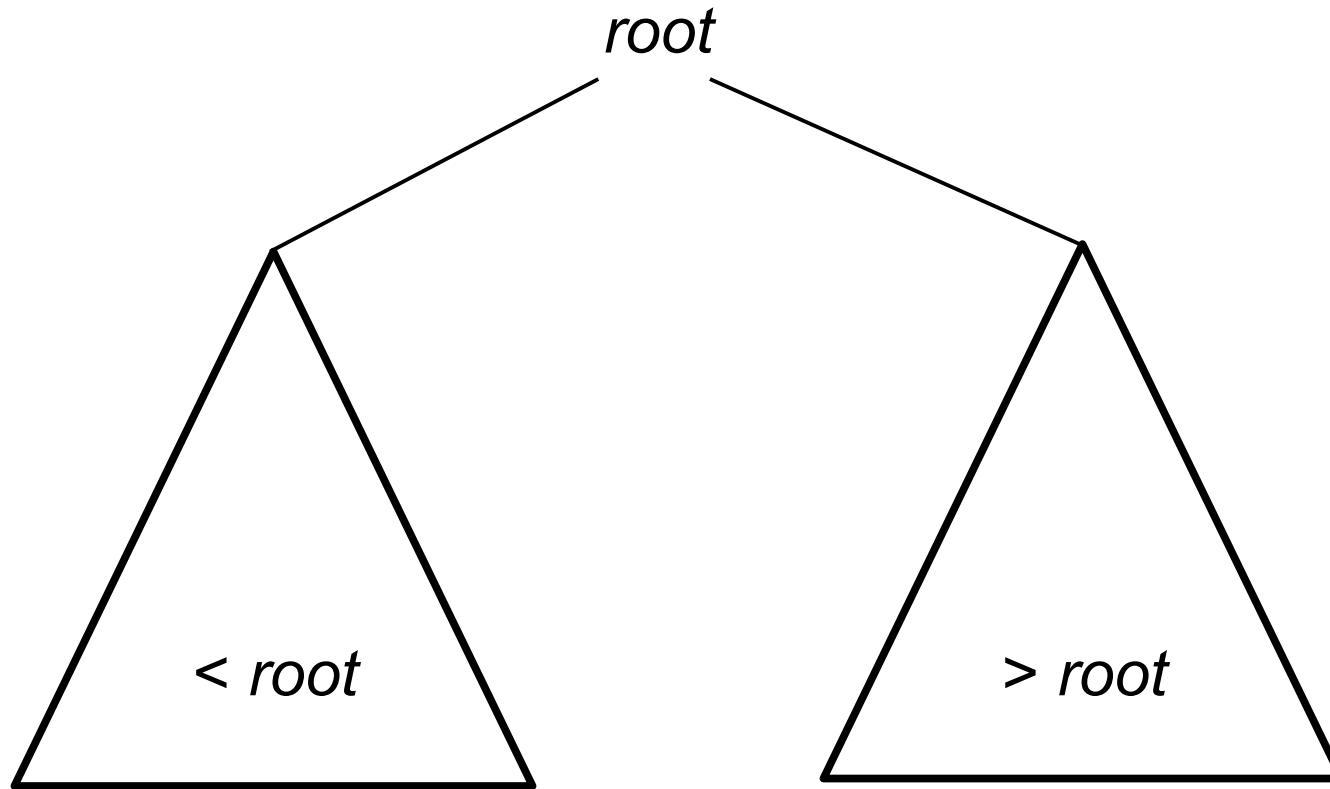
Balanced Binary Tree

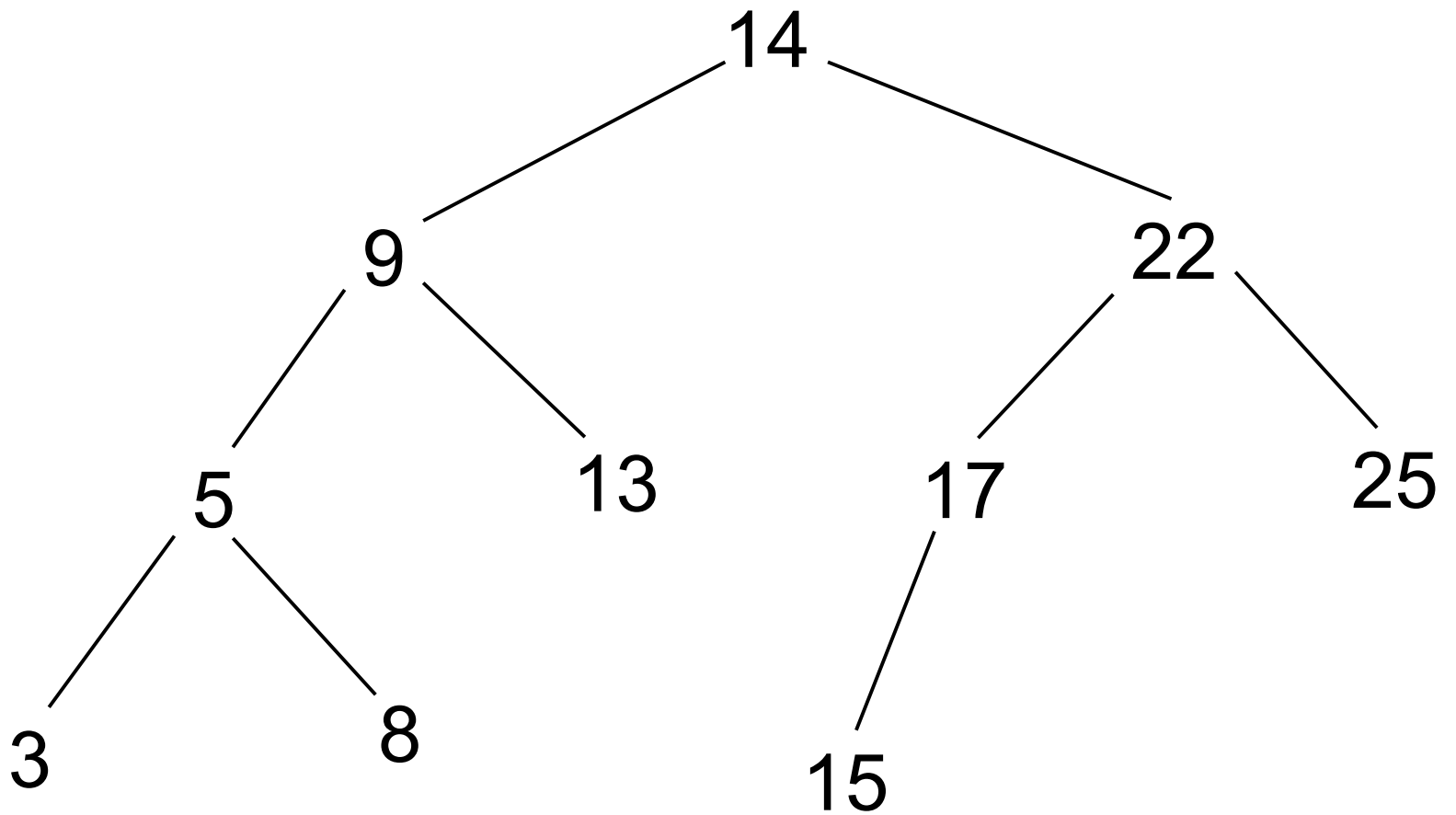


Unbalanced Binary Tree

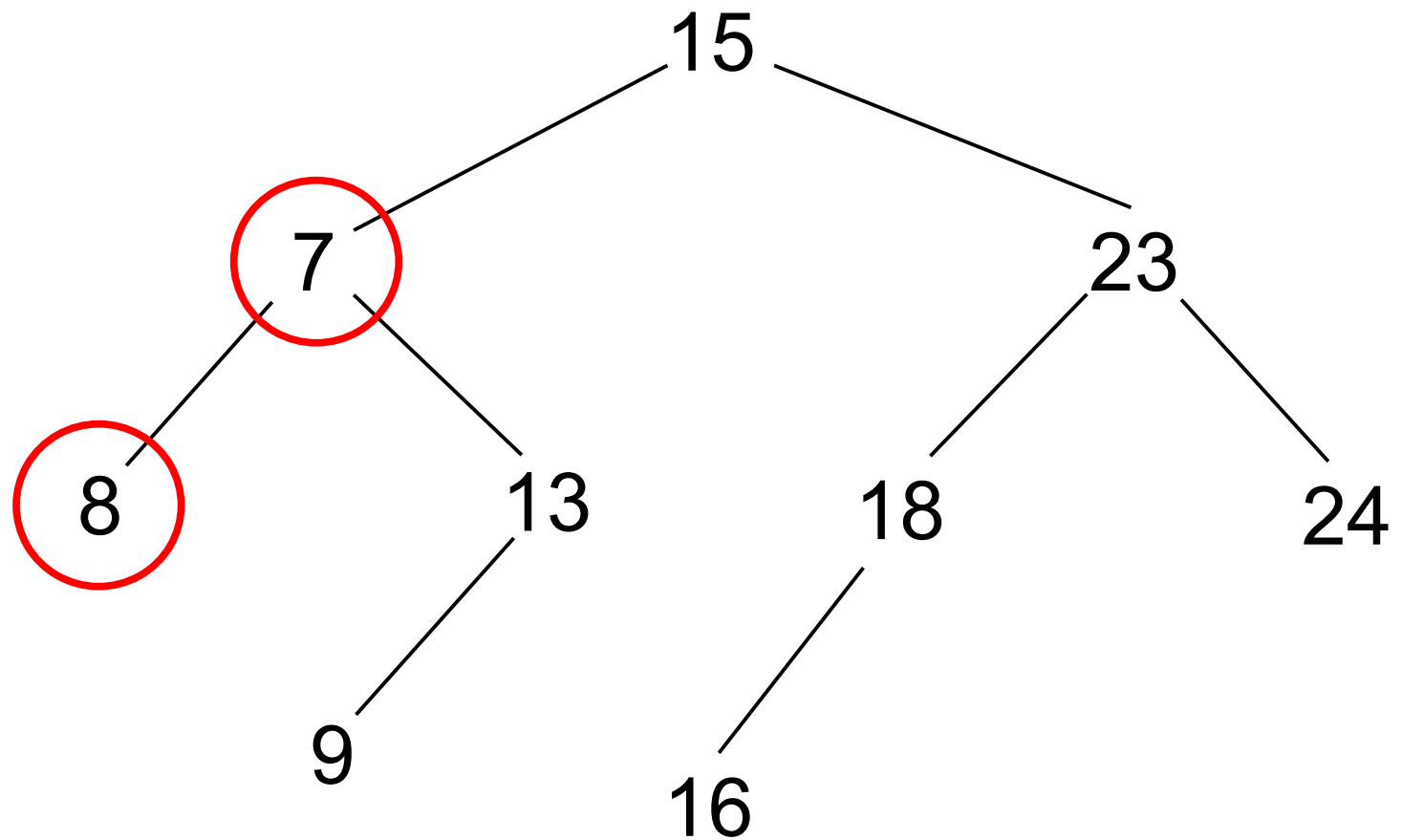


Binary Search Trees

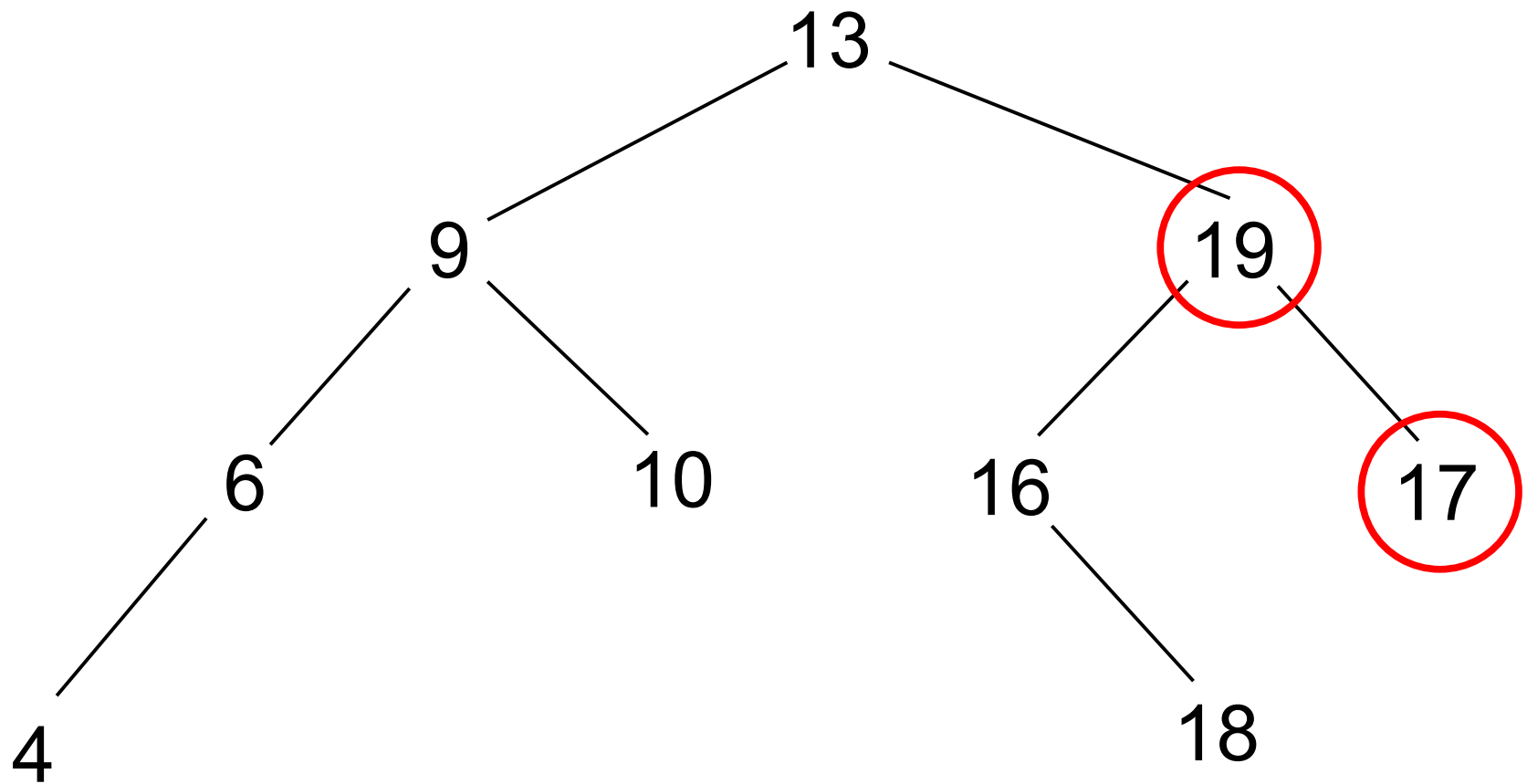




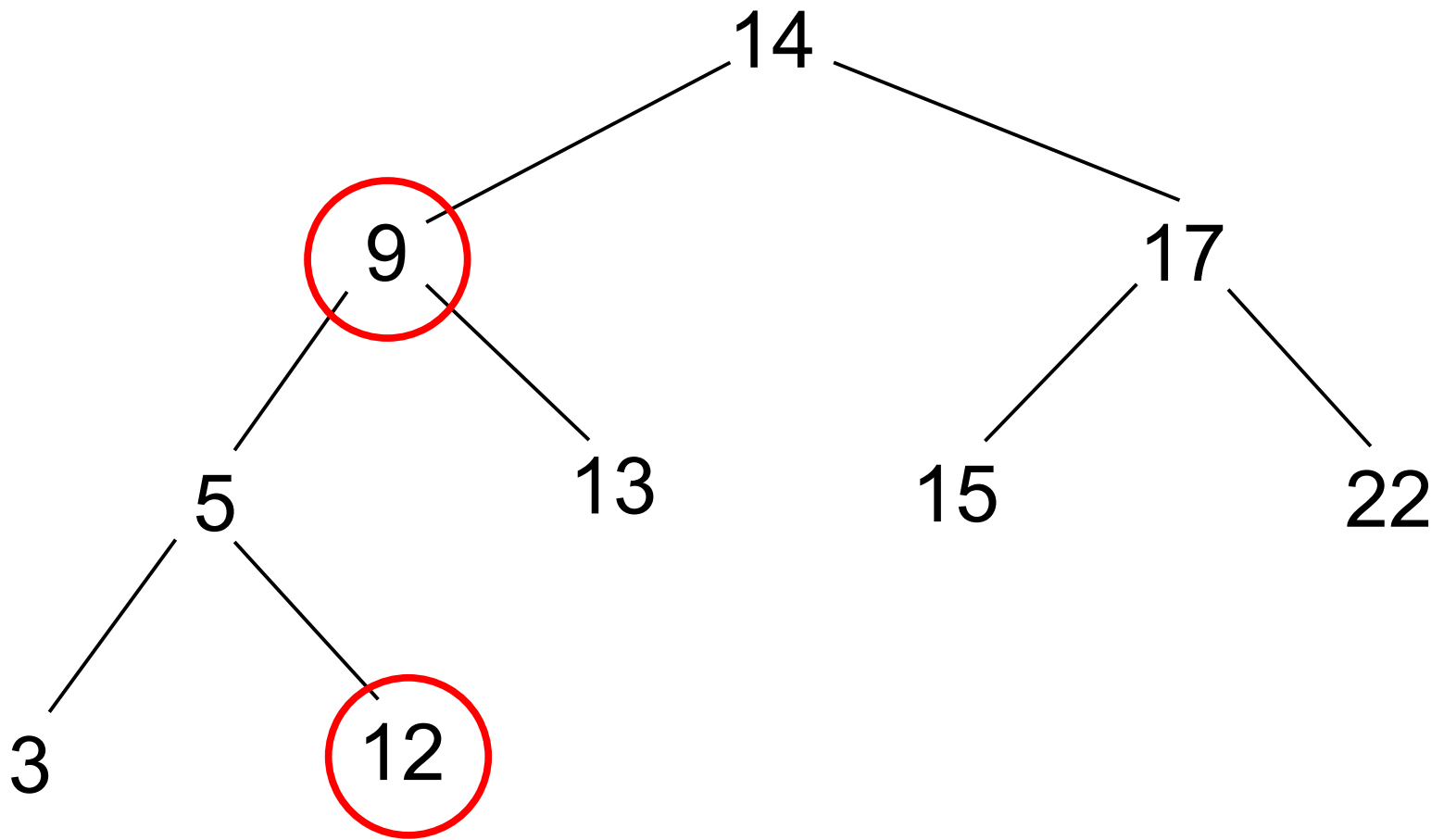
This is a valid binary search tree



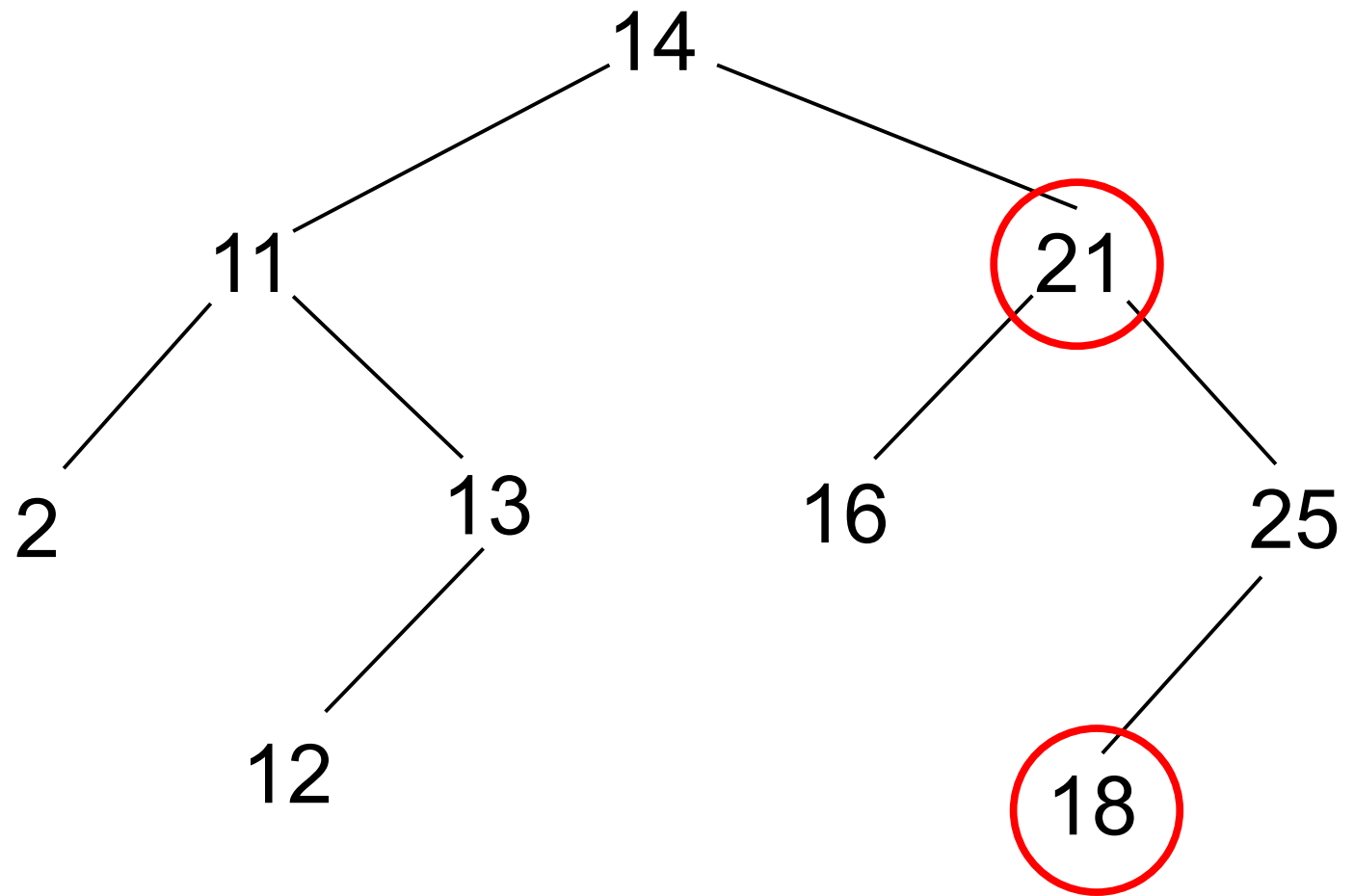
Violates BST property: 8 is not < 7



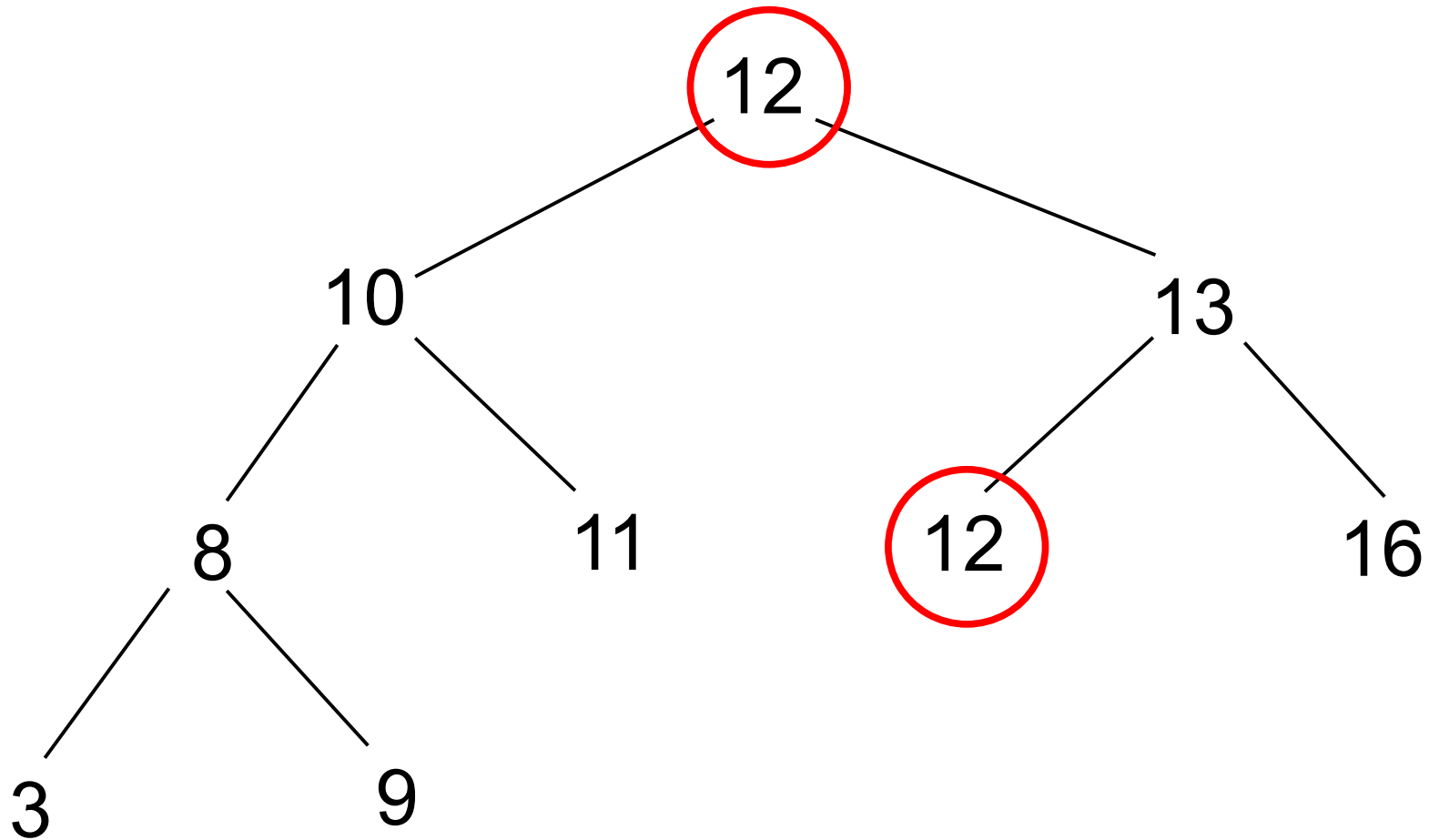
Violates BST property: 17 is not $>$ 19



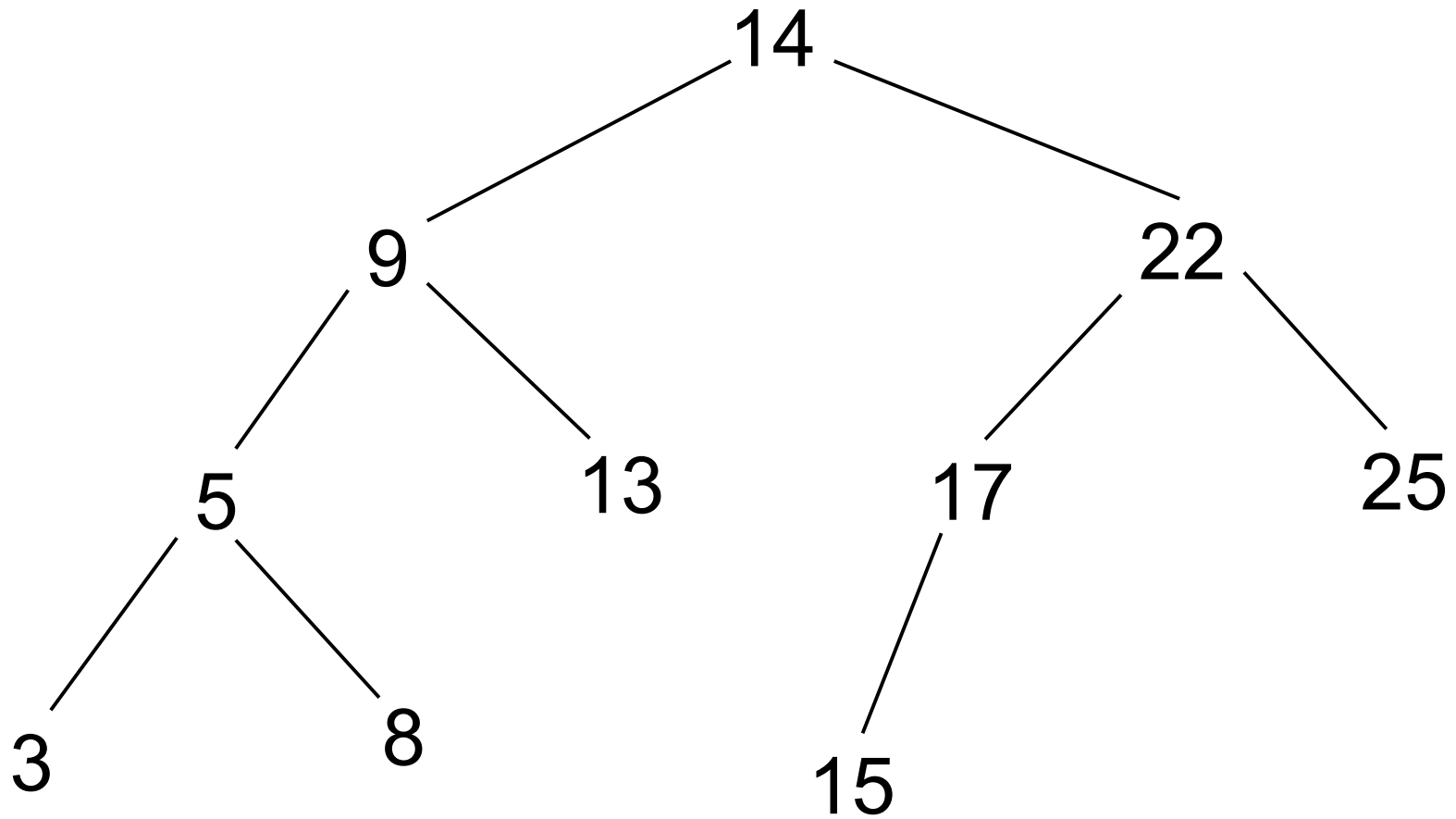
Violates BST property: 12 is not < 9



Violates BST property: 18 is not $>$ 21



Duplicate elements are not allowed

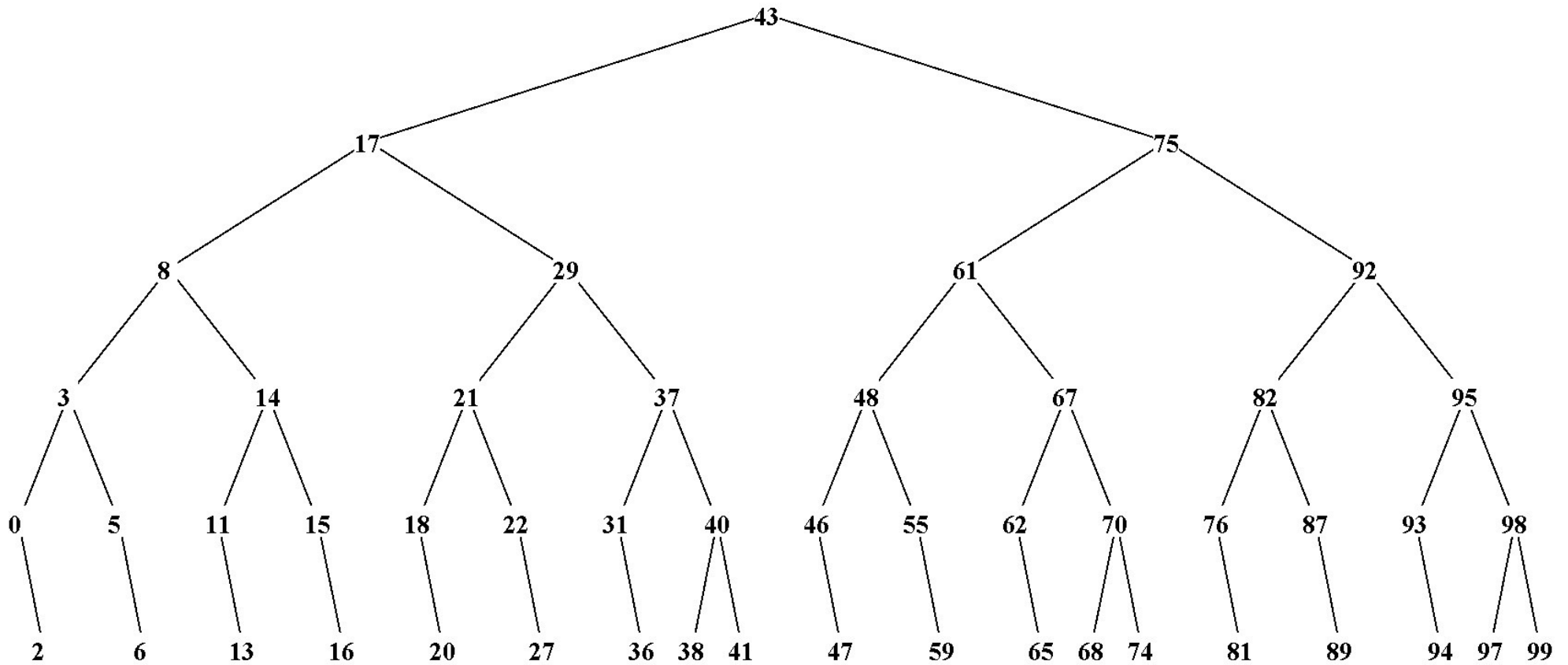


Binary search trees are good at representing **sets**

{ 3, 5, 8, 9, 13, 14, 15, 17, 22, 25 }

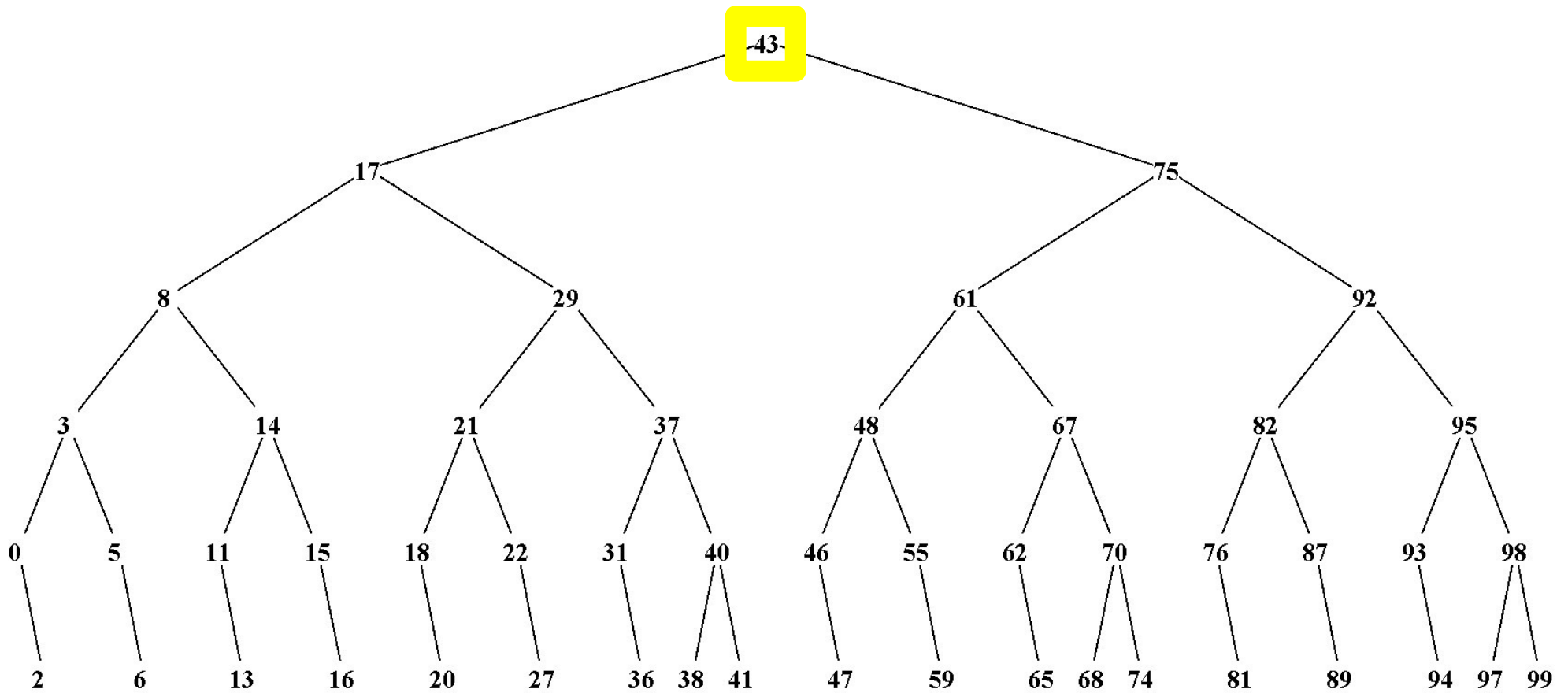
Testing for Membership

is 65 a member?



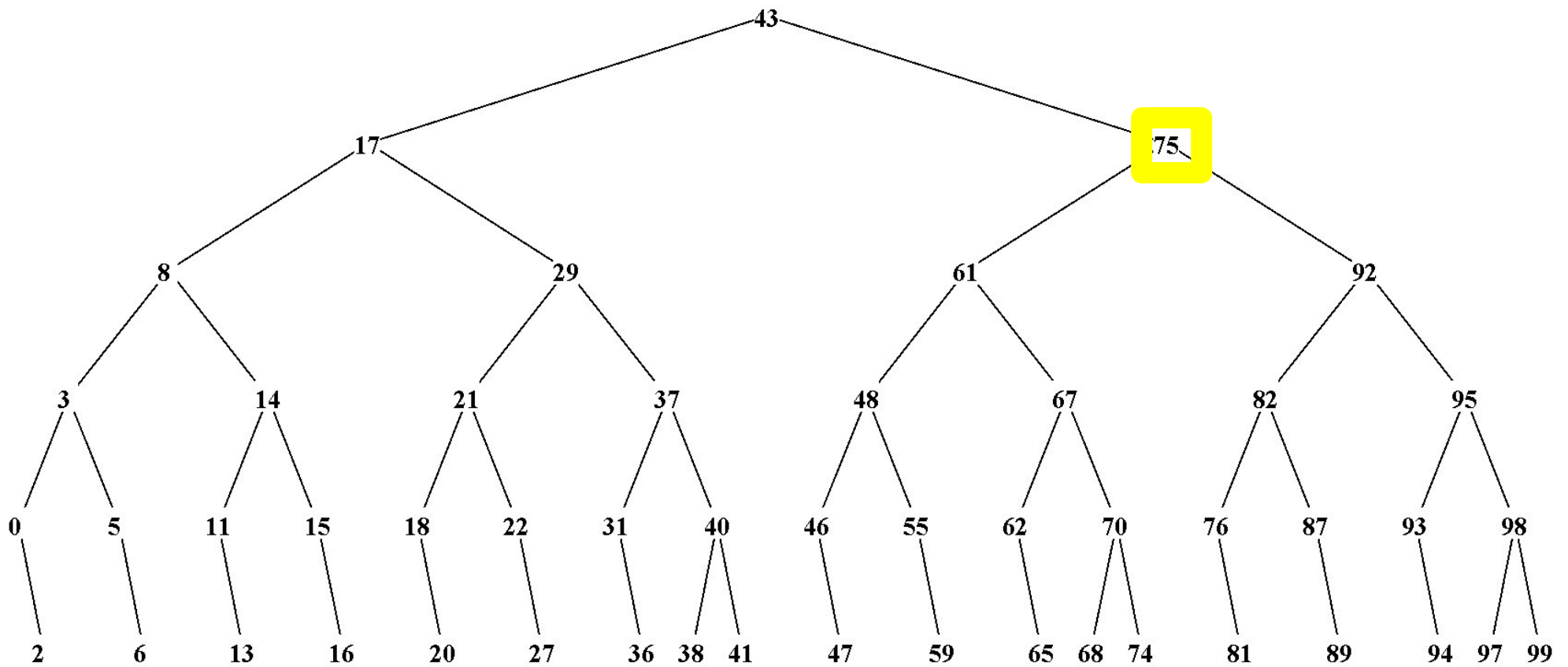
Testing for Membership

is 65 a member?



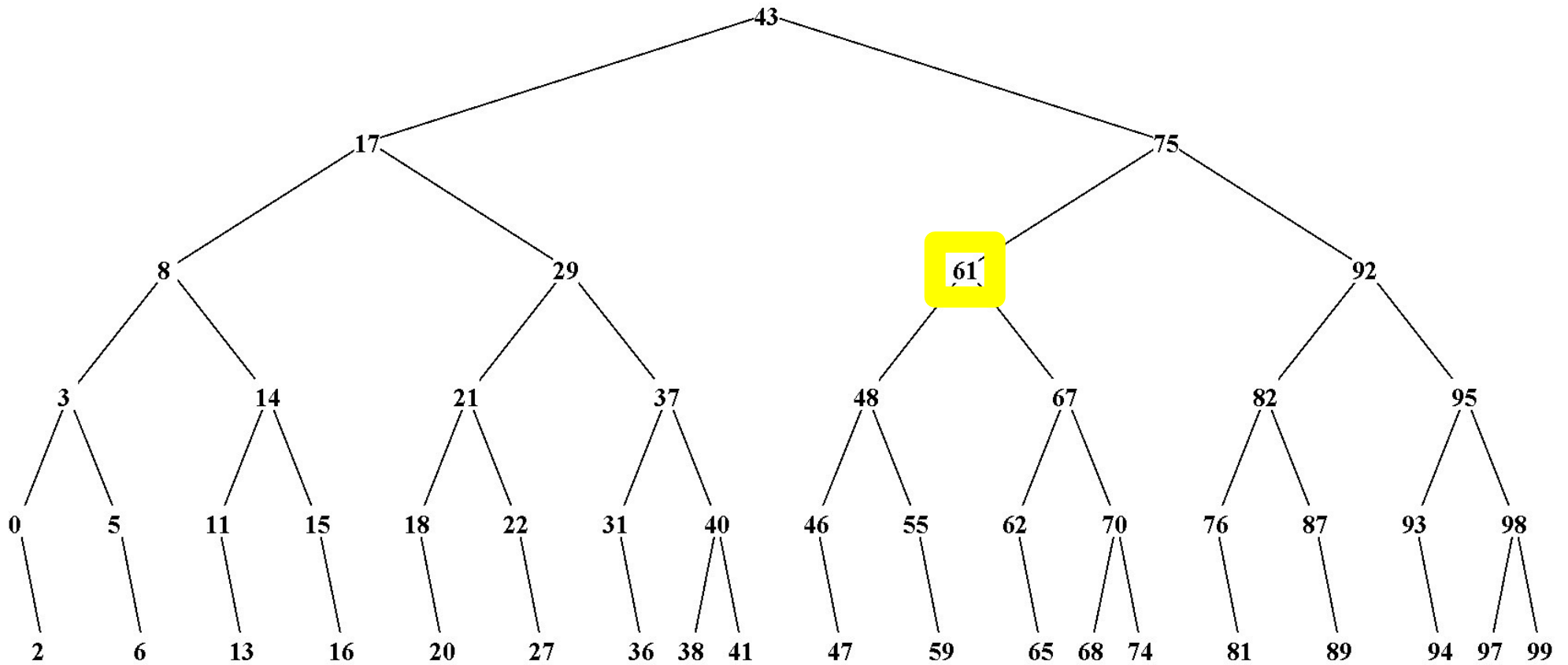
Testing for Membership

is 65 a member?



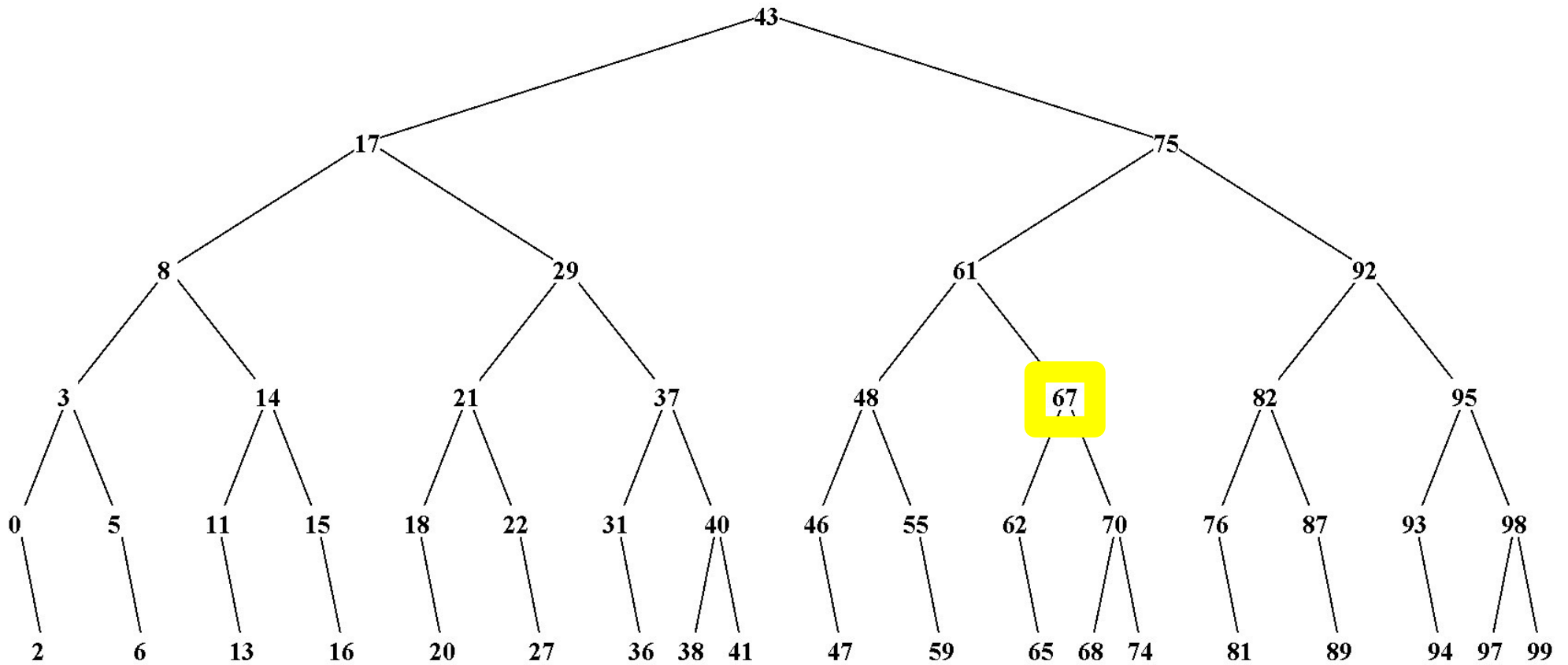
Testing for Membership

is 65 a member?



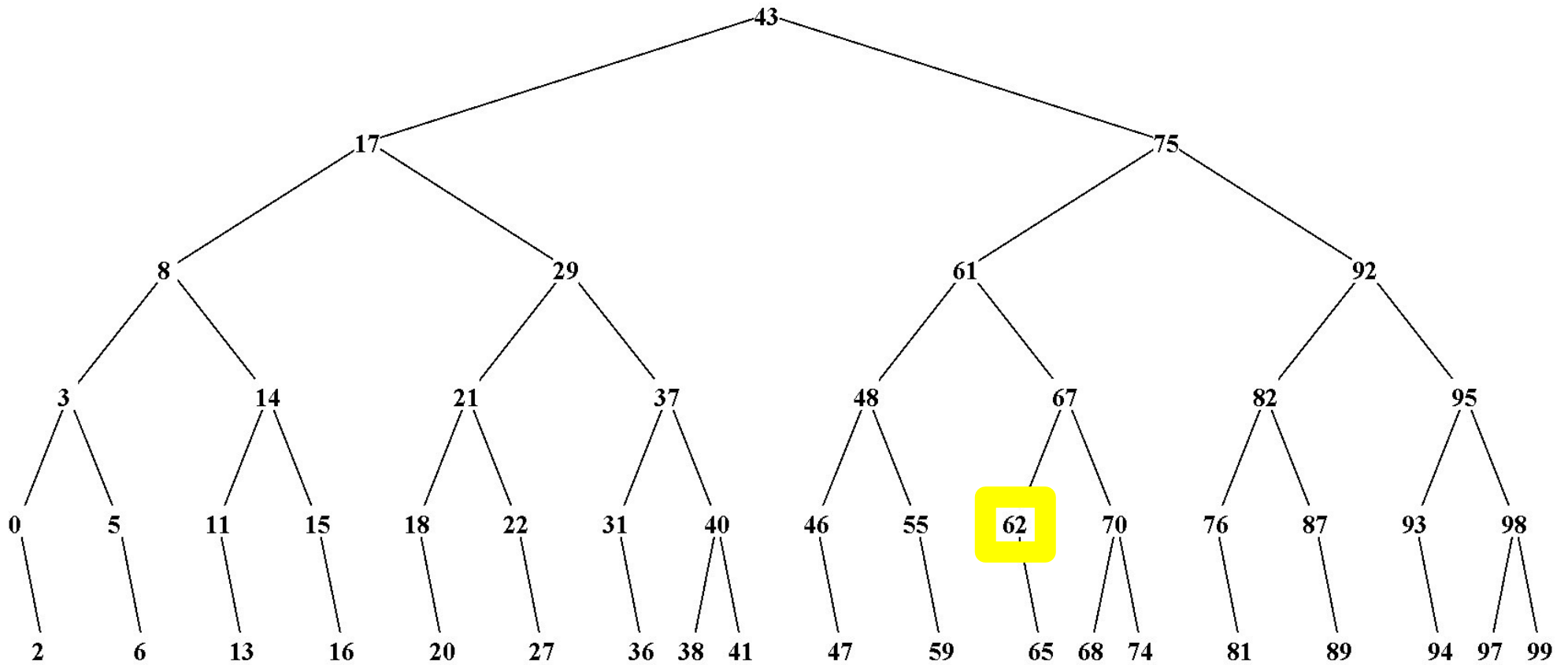
Testing for Membership

is 65 a member?



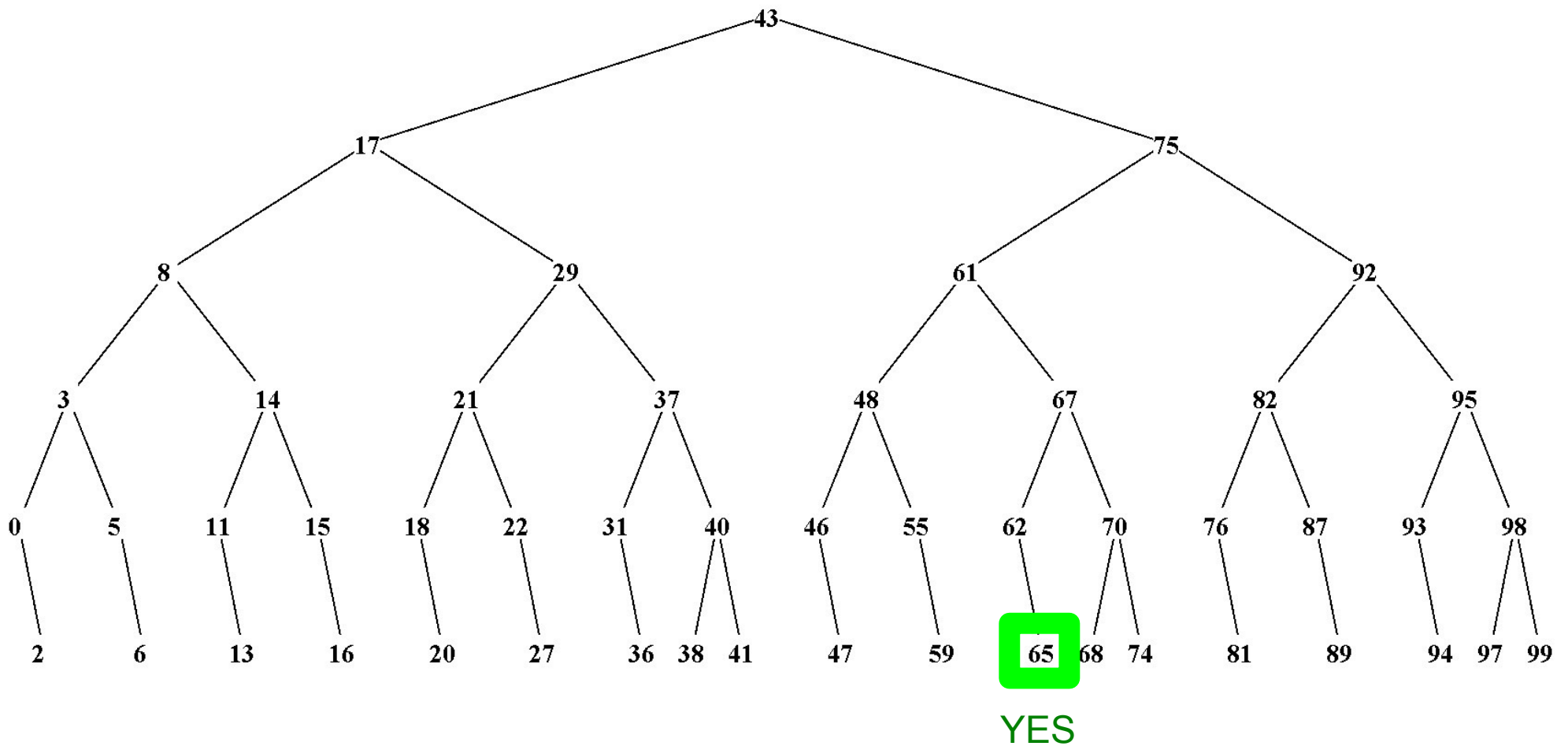
Testing for Membership

is 65 a member?



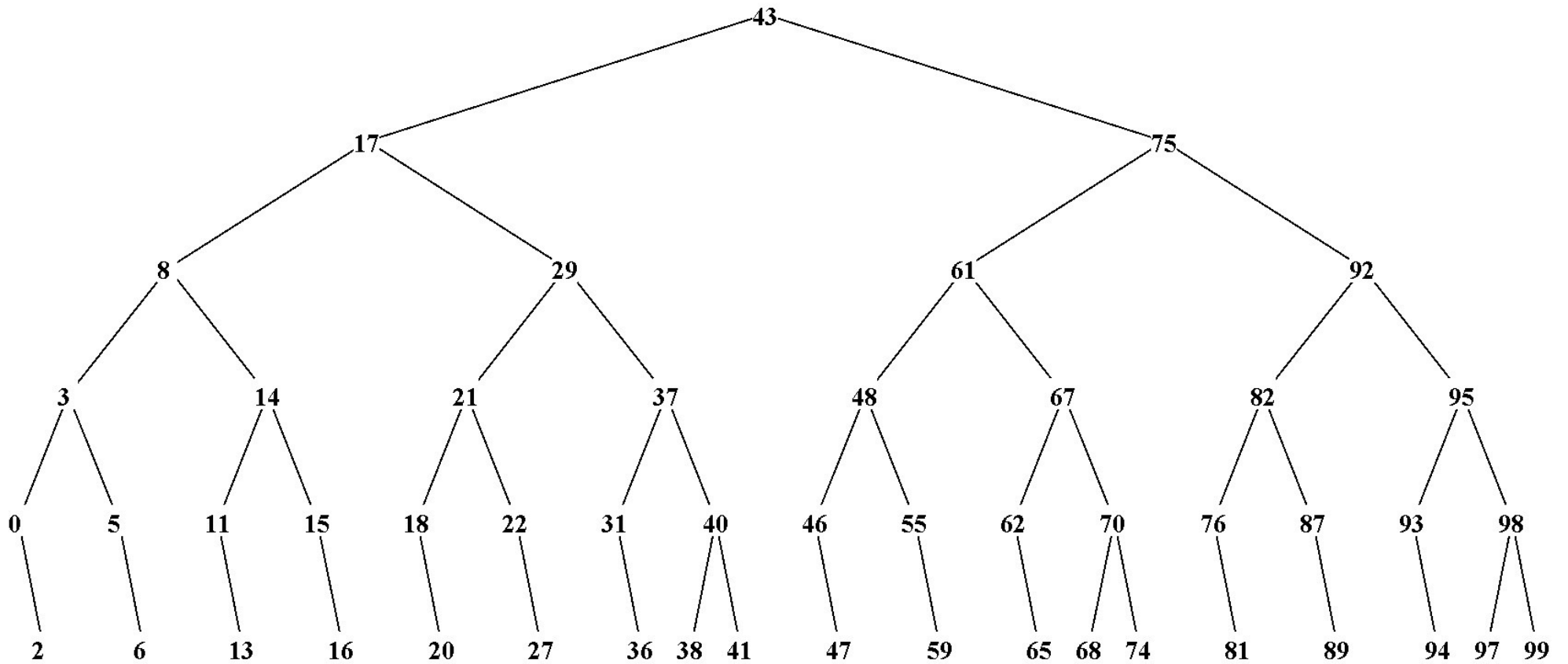
Testing for Membership

is 65 a member?



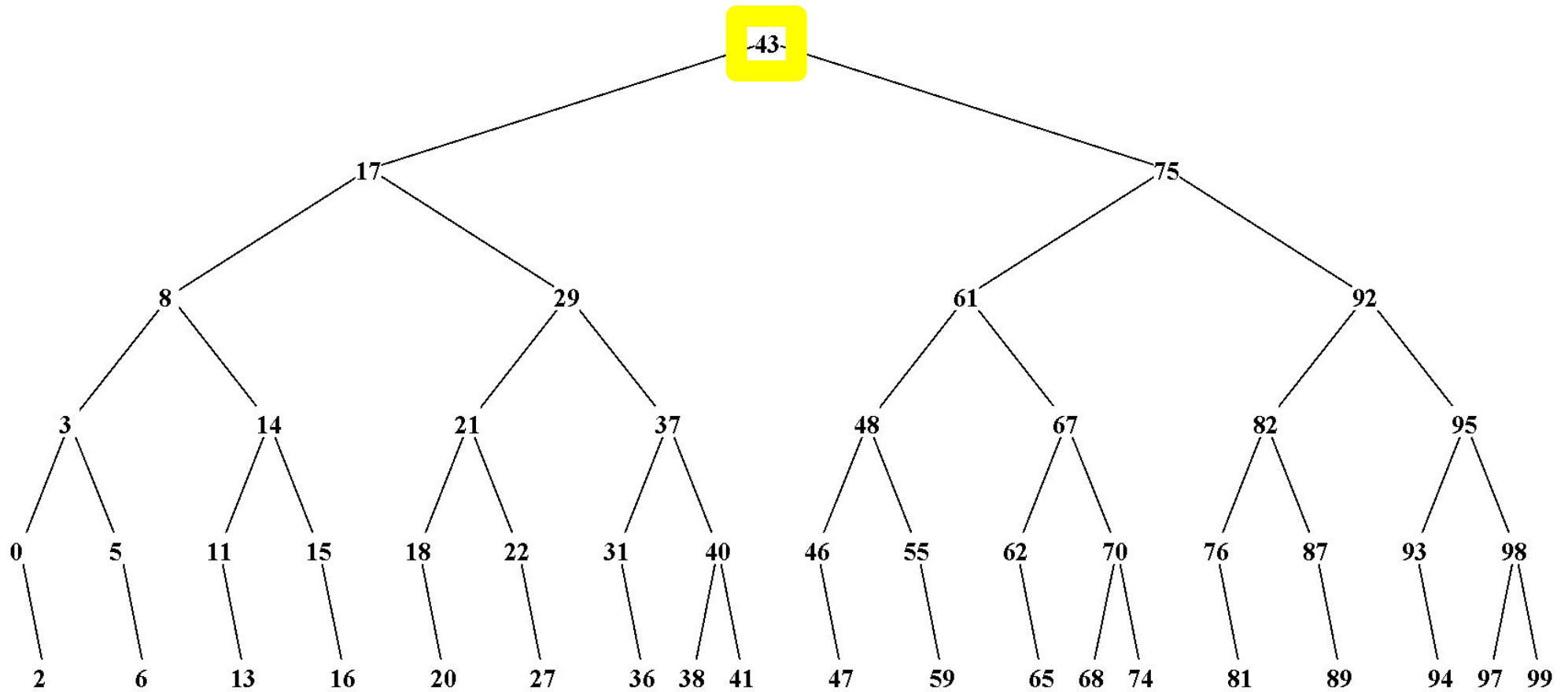
Testing for Membership

is 12 a member?



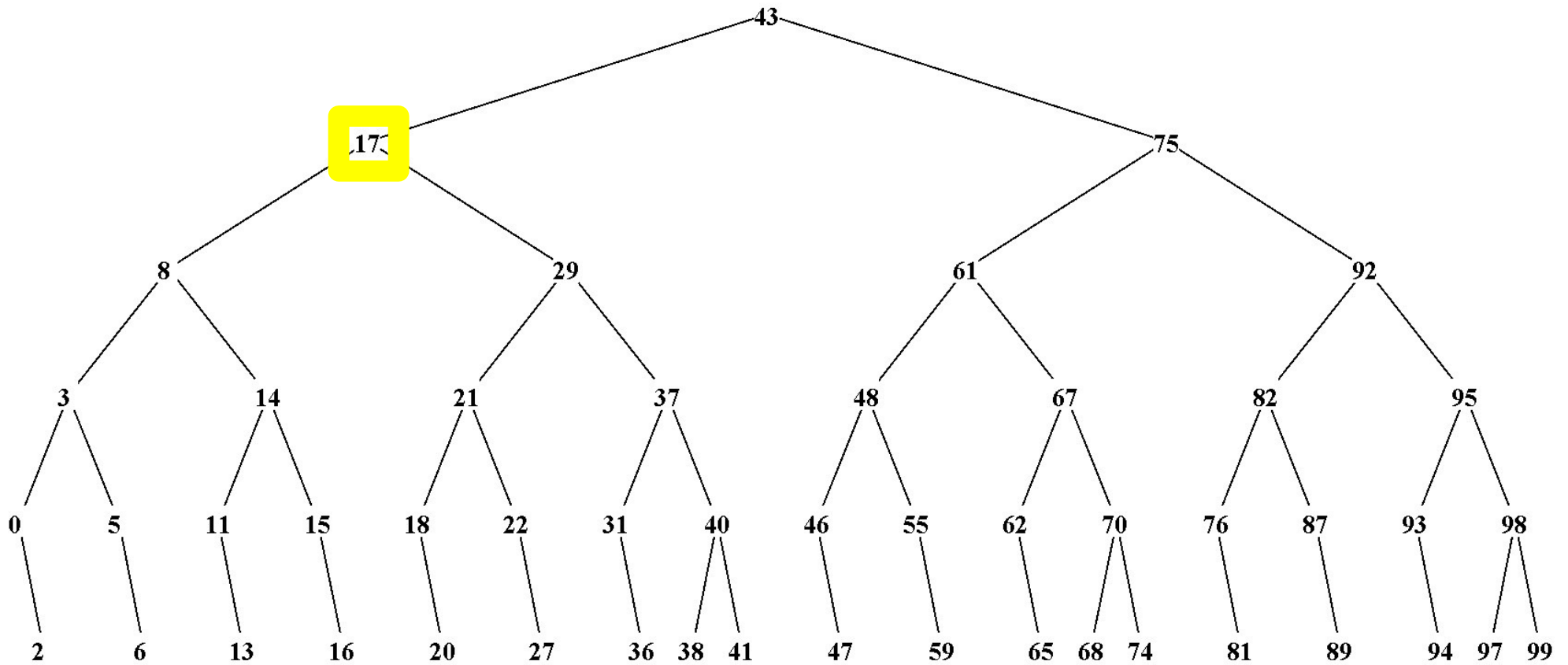
Testing for Membership

is 12 a member?



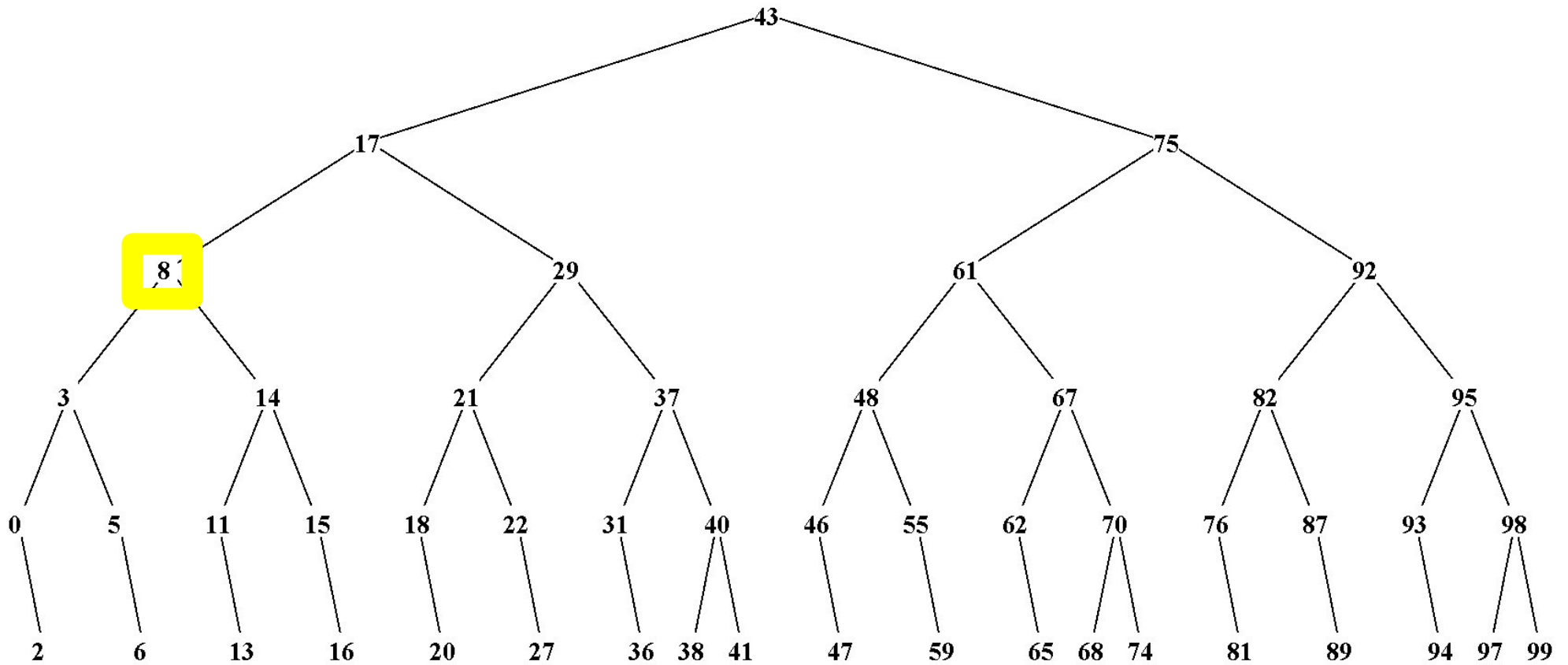
Testing for Membership

is 12 a member?



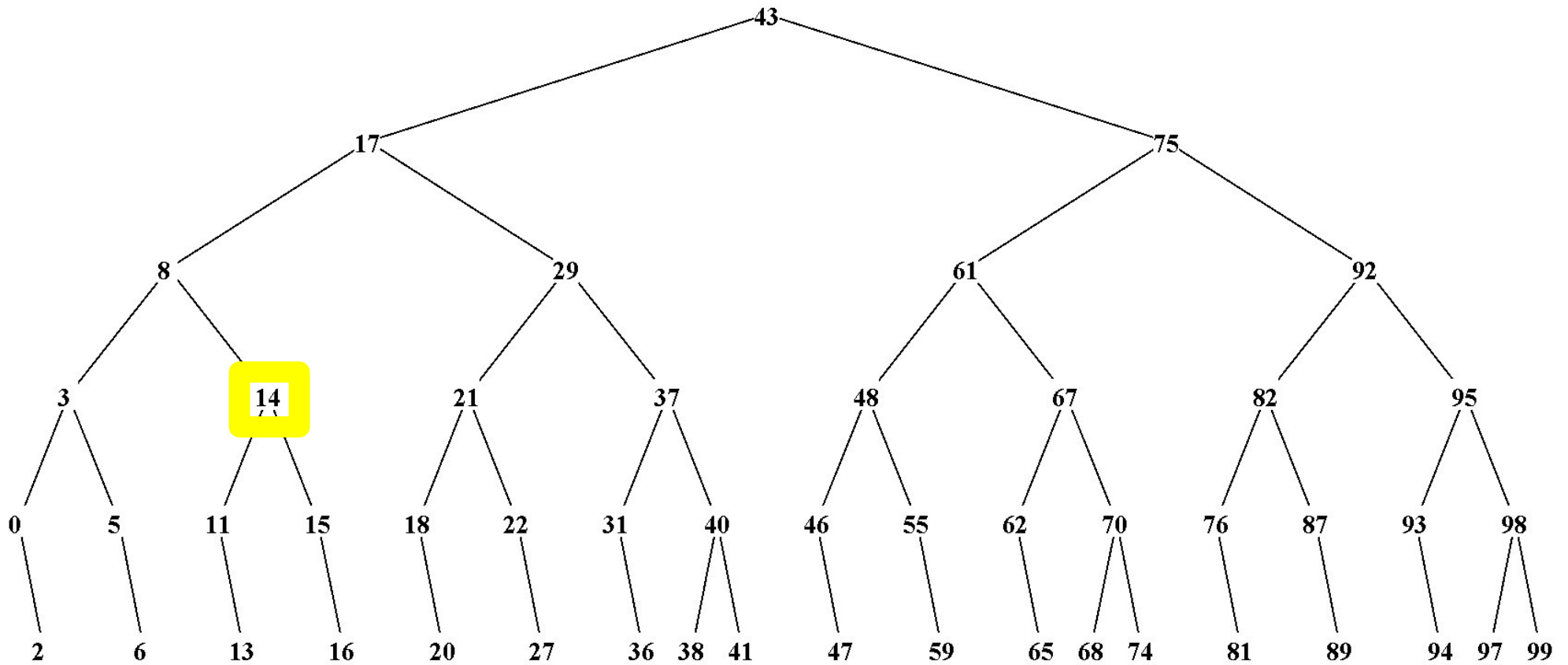
Testing for Membership

is 12 a member?



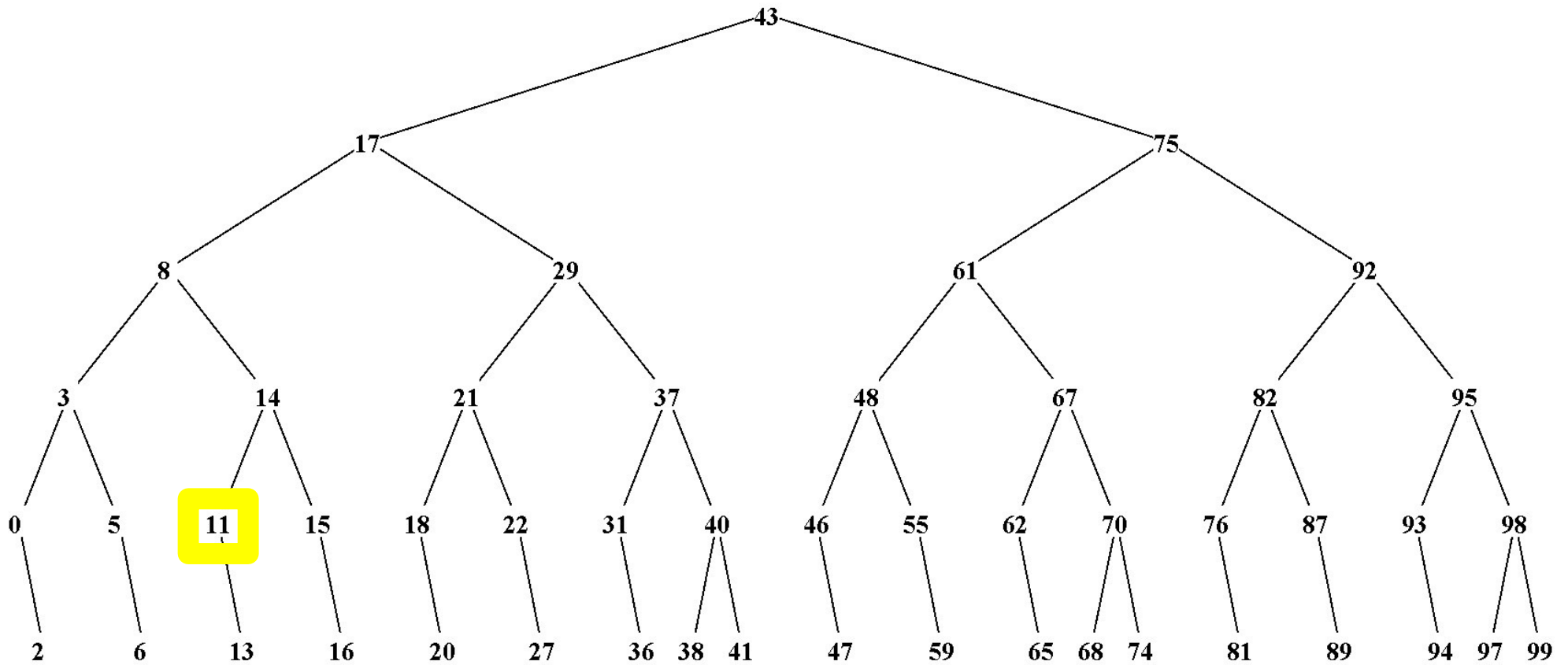
Testing for Membership

is 12 a member?



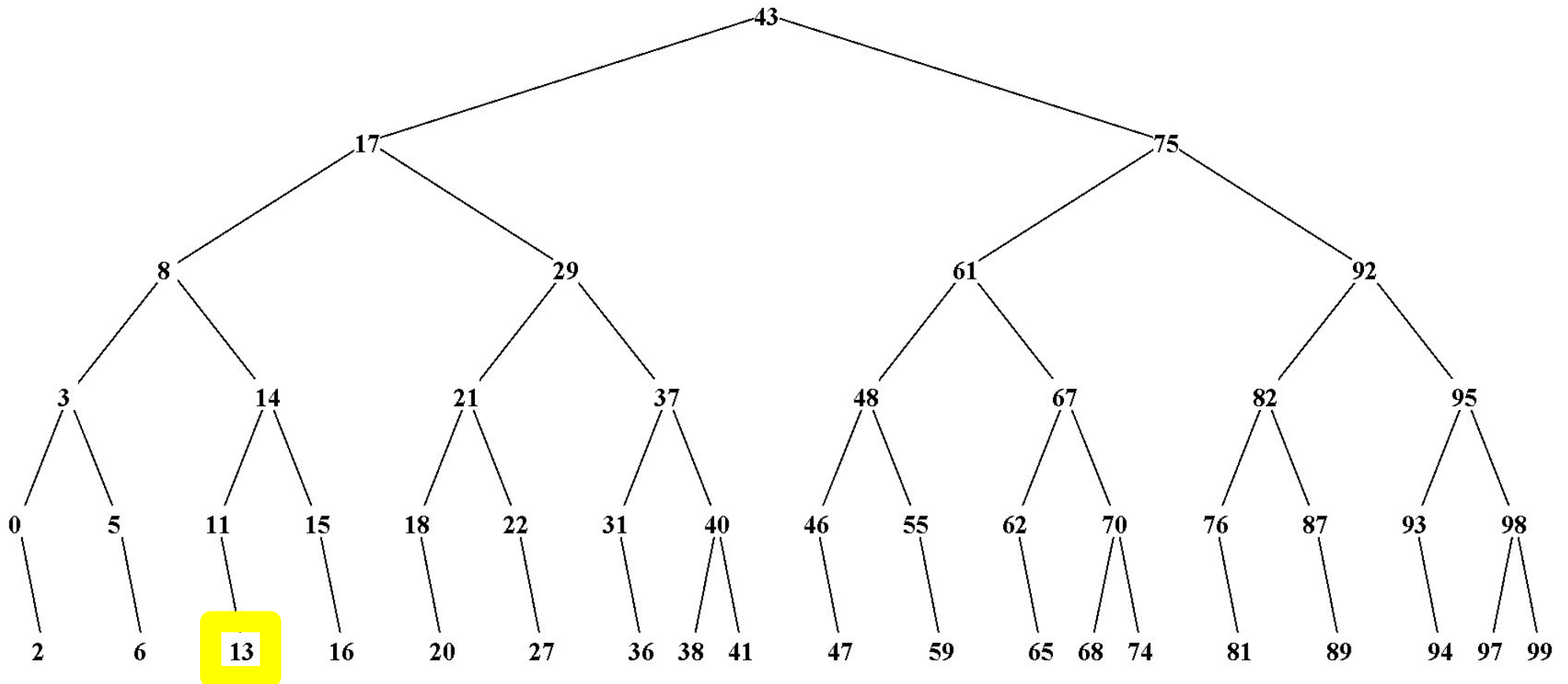
Testing for Membership

is 12 a member?



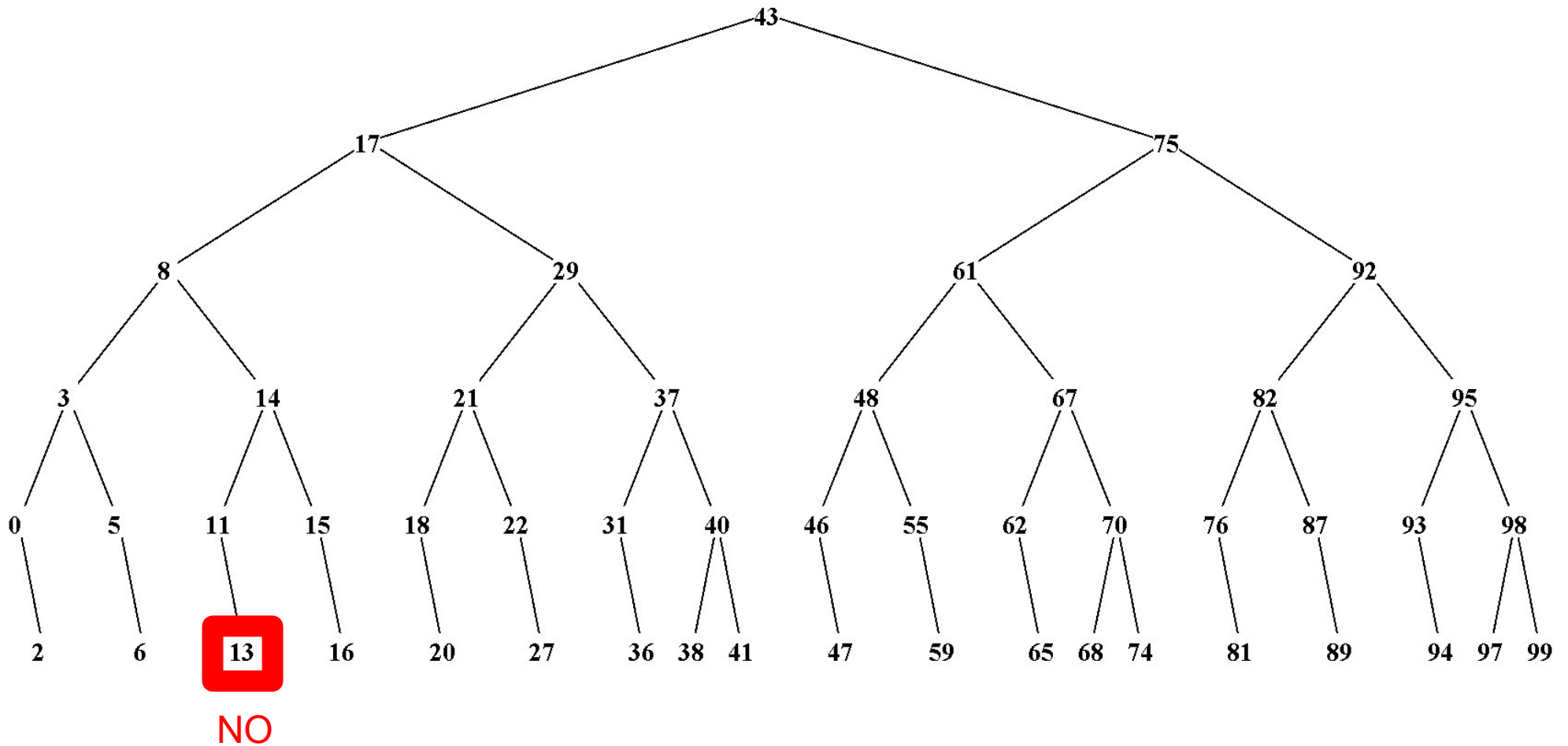
Testing for Membership

is 12 a member?



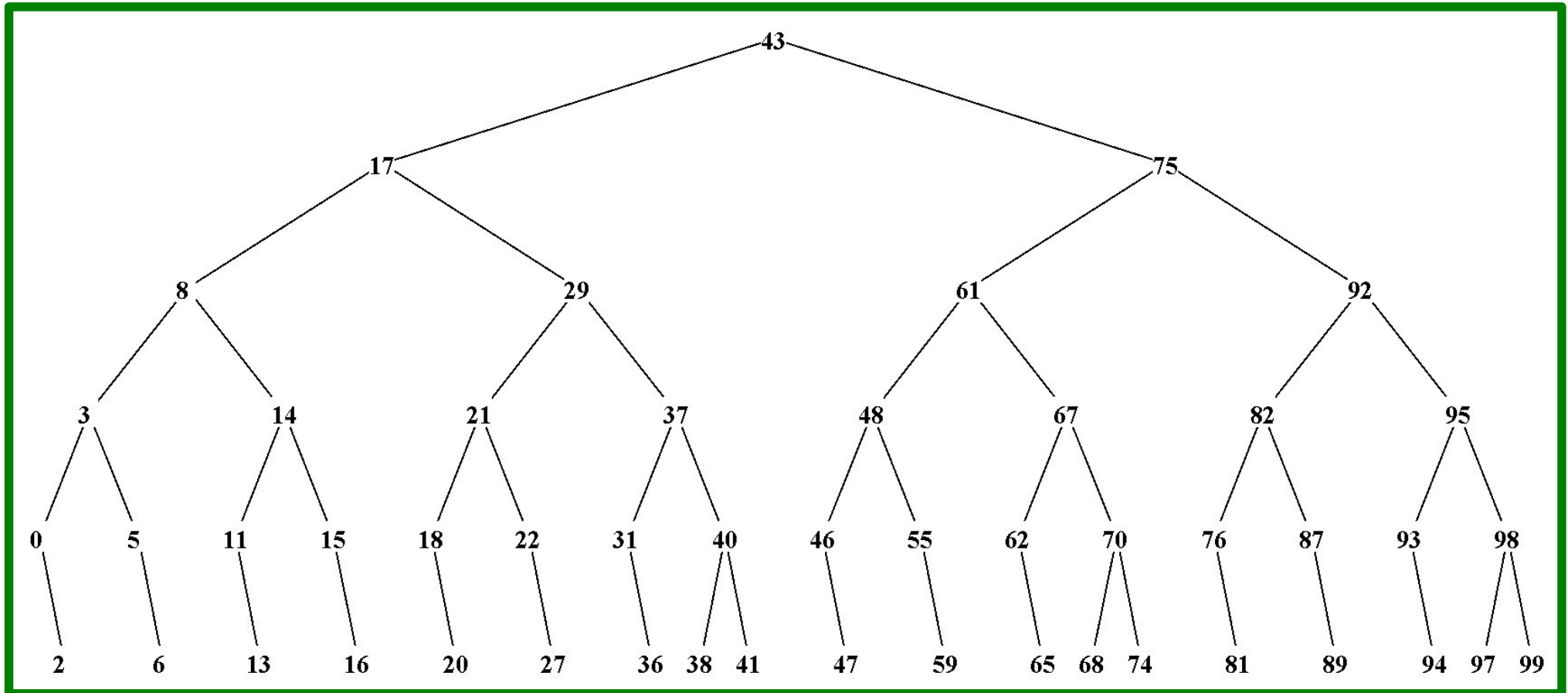
Testing for Membership

is 12 a member?



If the Tree is Balanced ...

n elements

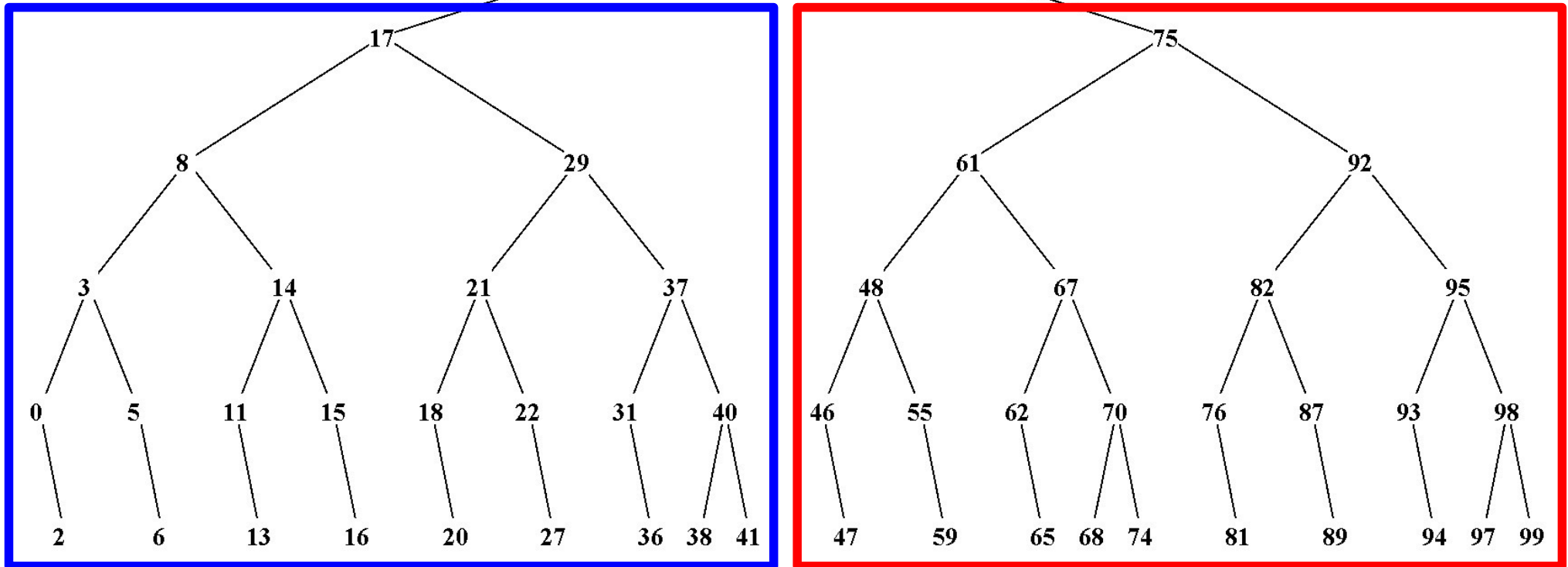


If the Tree is Balanced ...

n elements

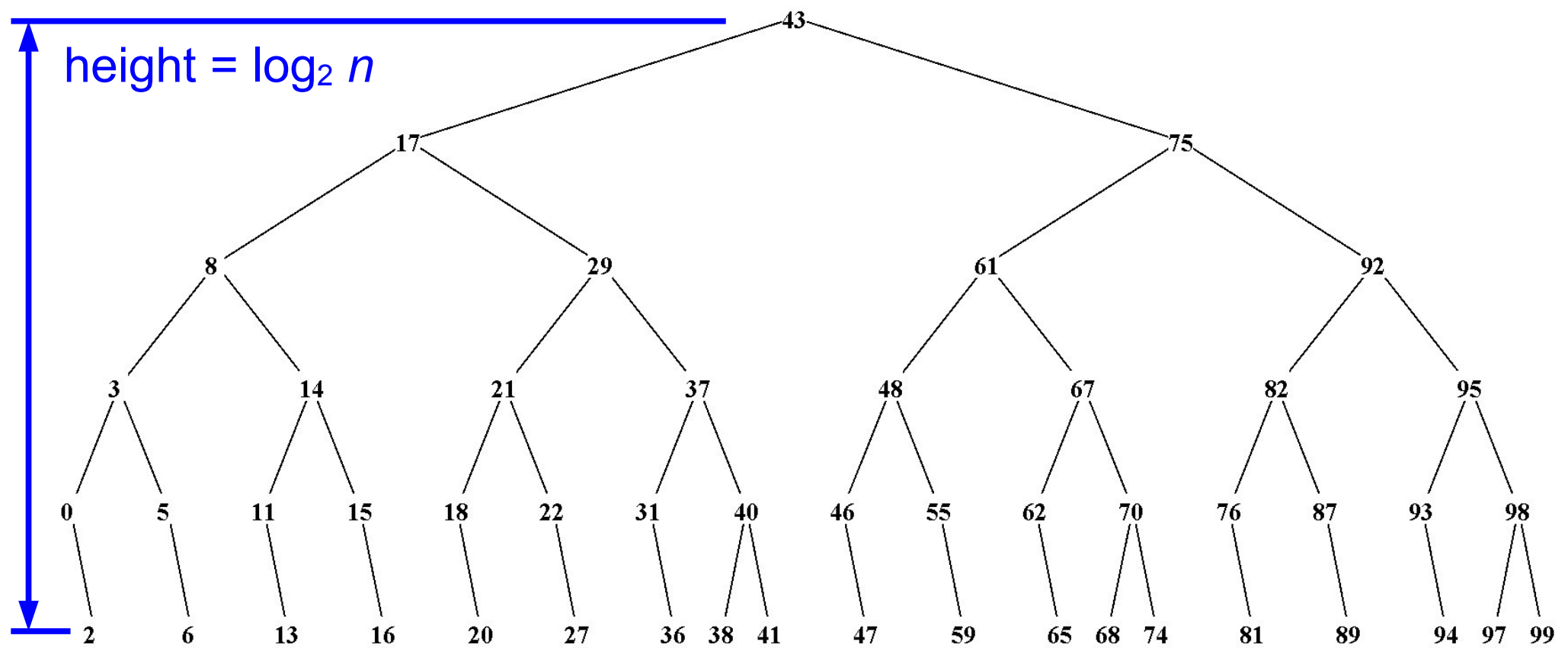
about $n / 2$ elements

about $n / 2$ elements



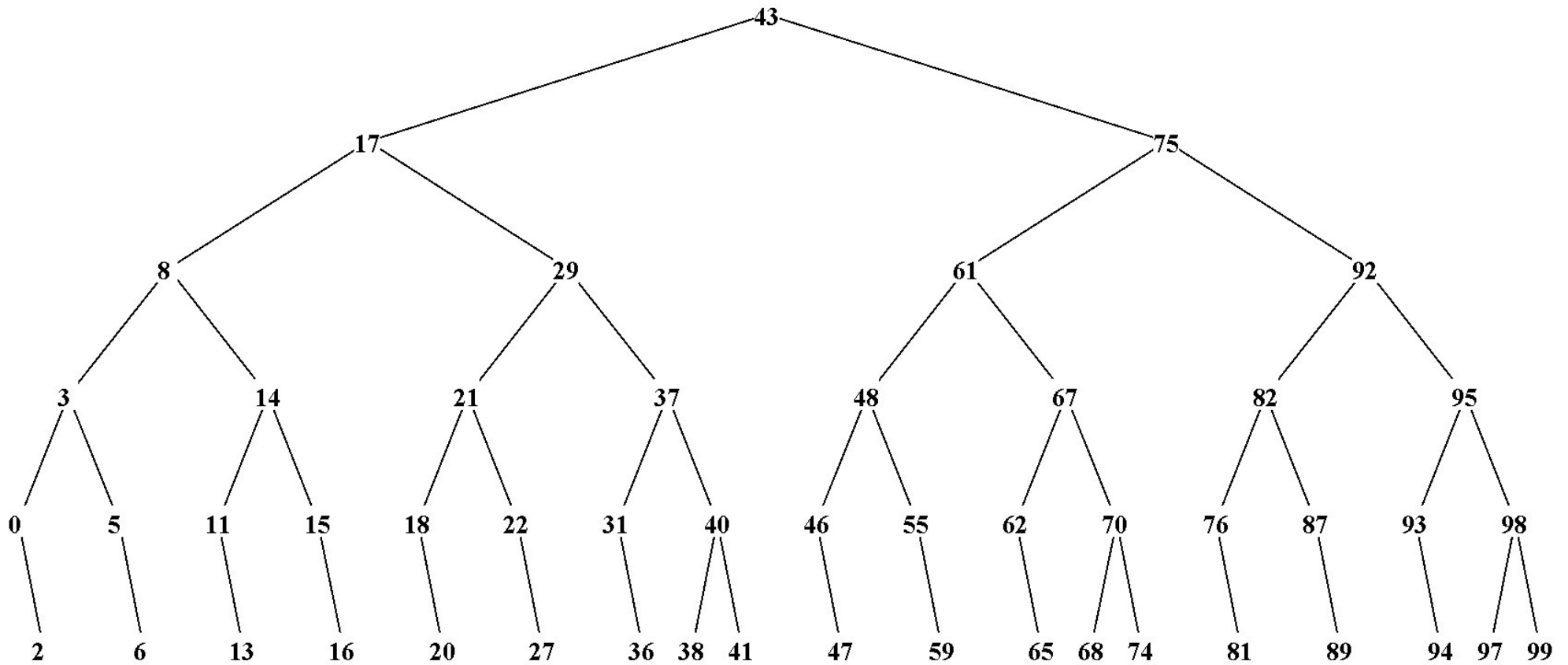
If the Tree is Balanced ...

n elements



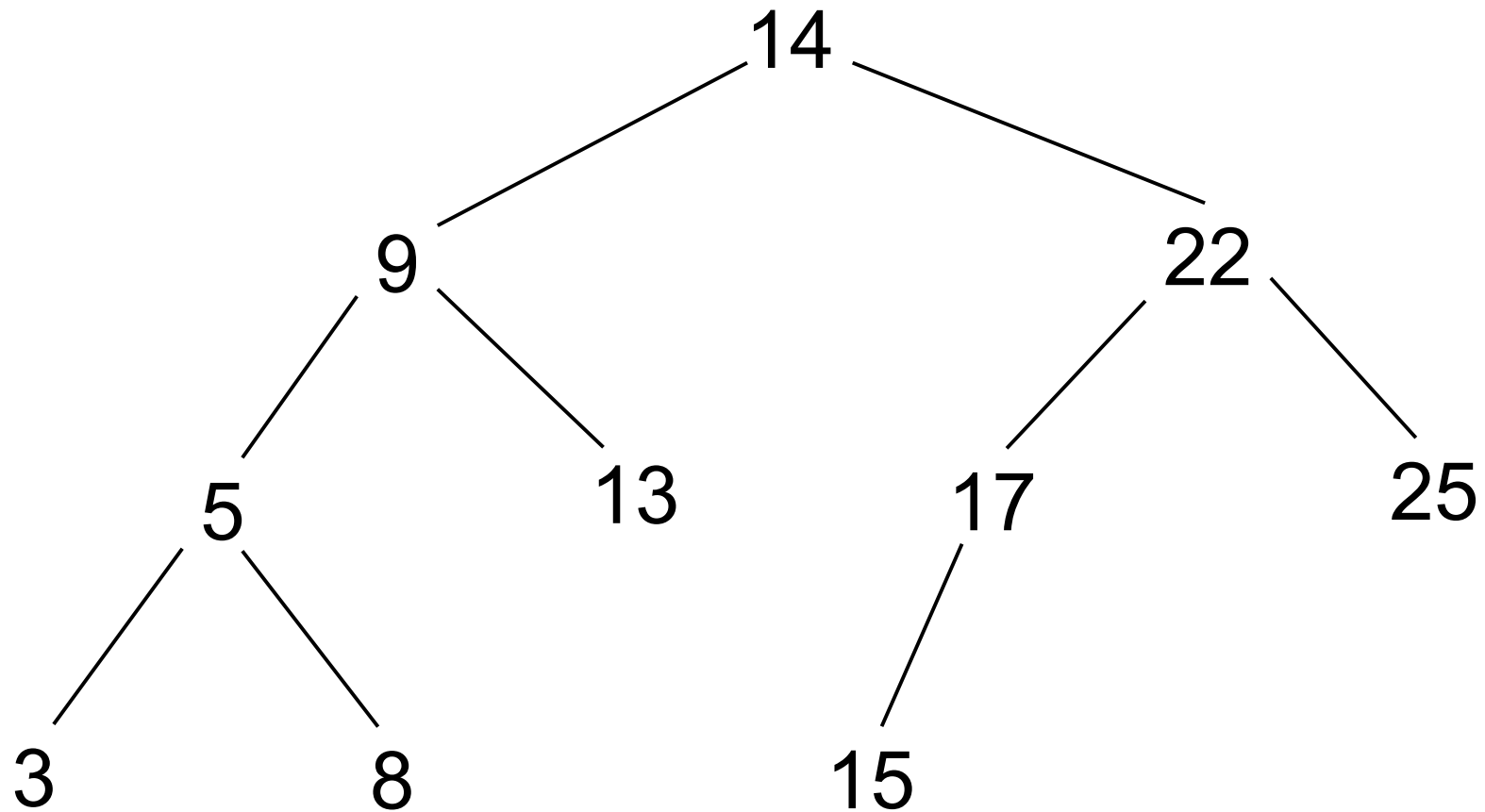
Testing for Membership

$O(\log n)$ time complexity

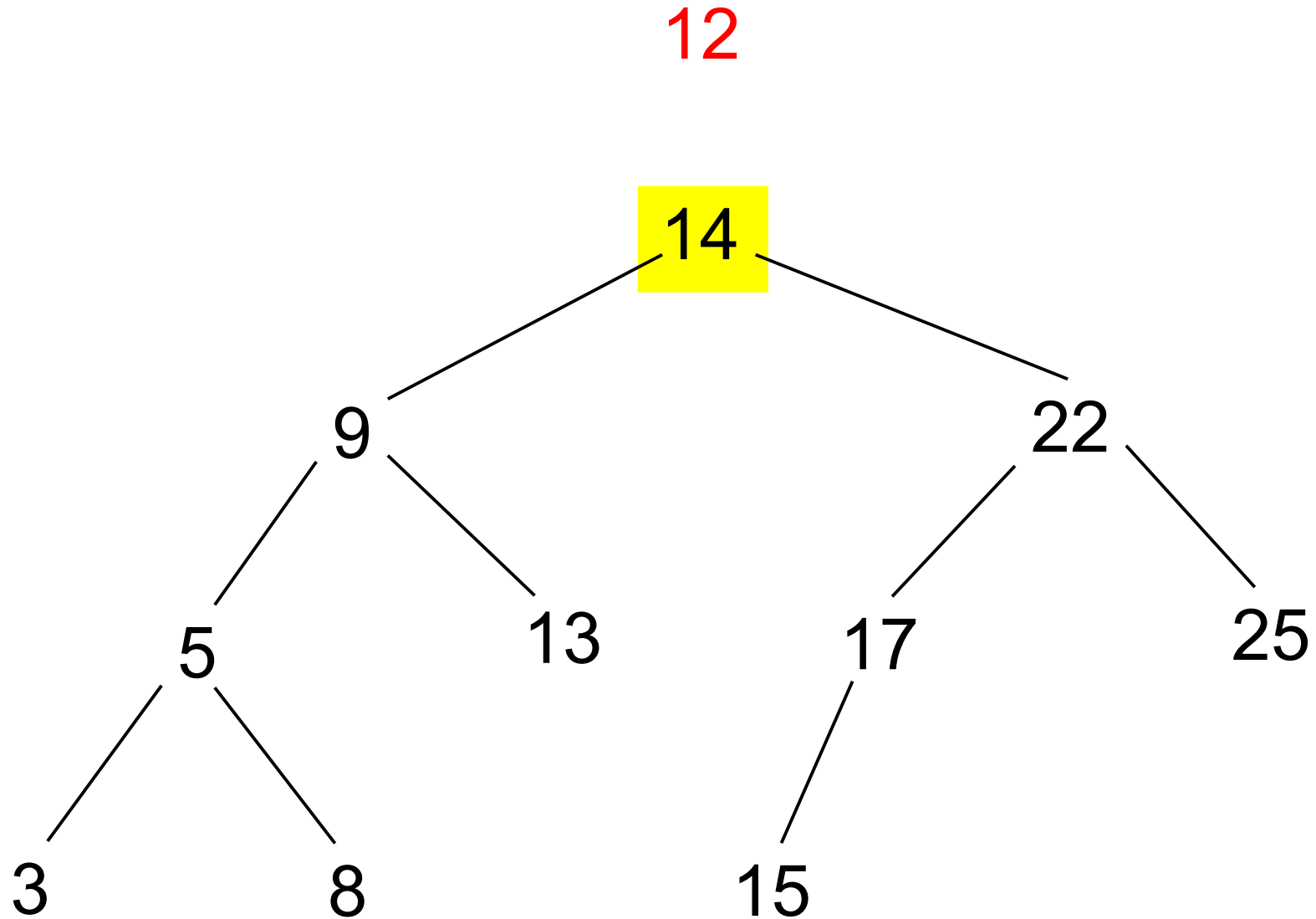


Inserting a New Element

12

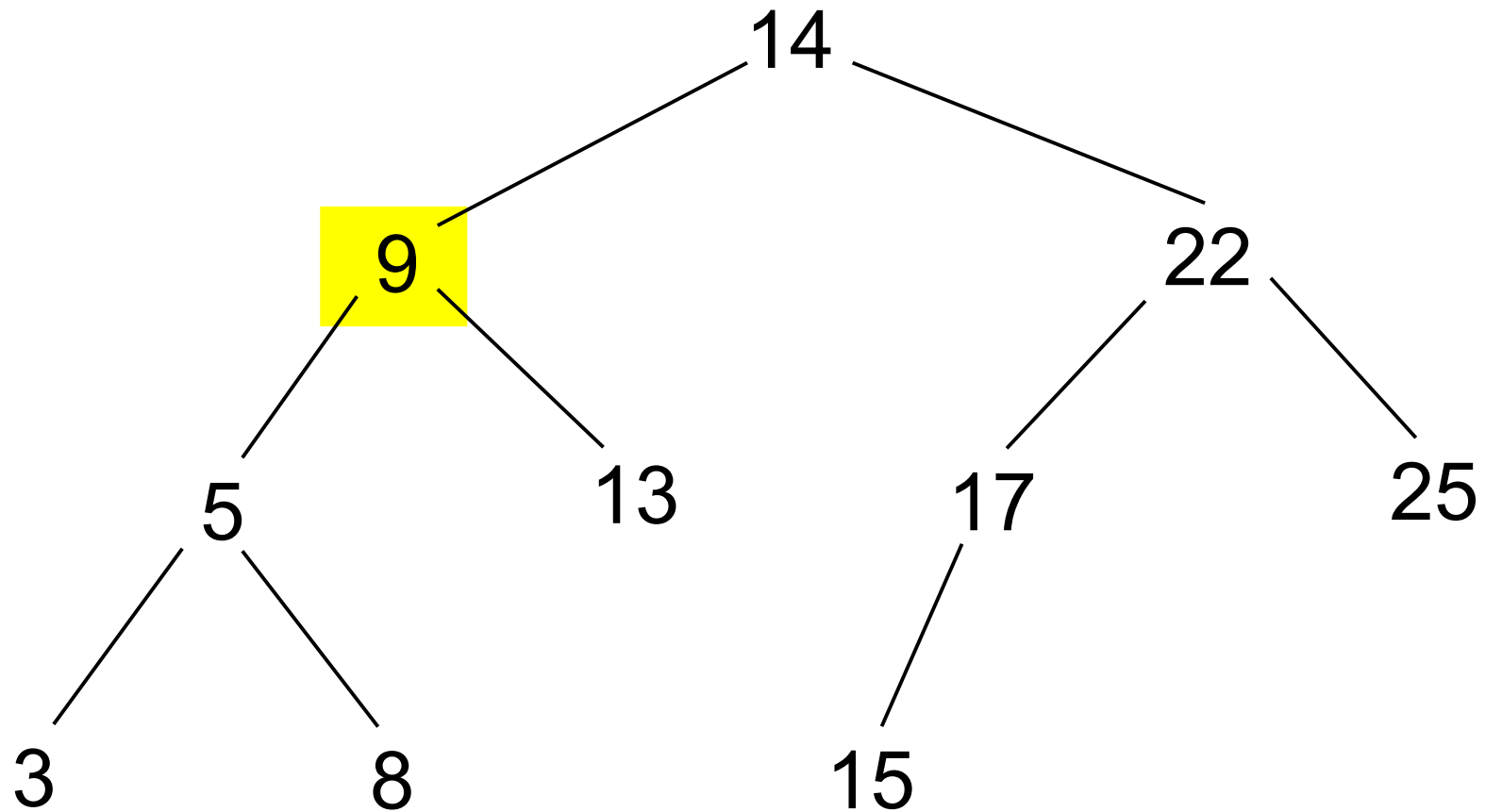


Inserting a New Element

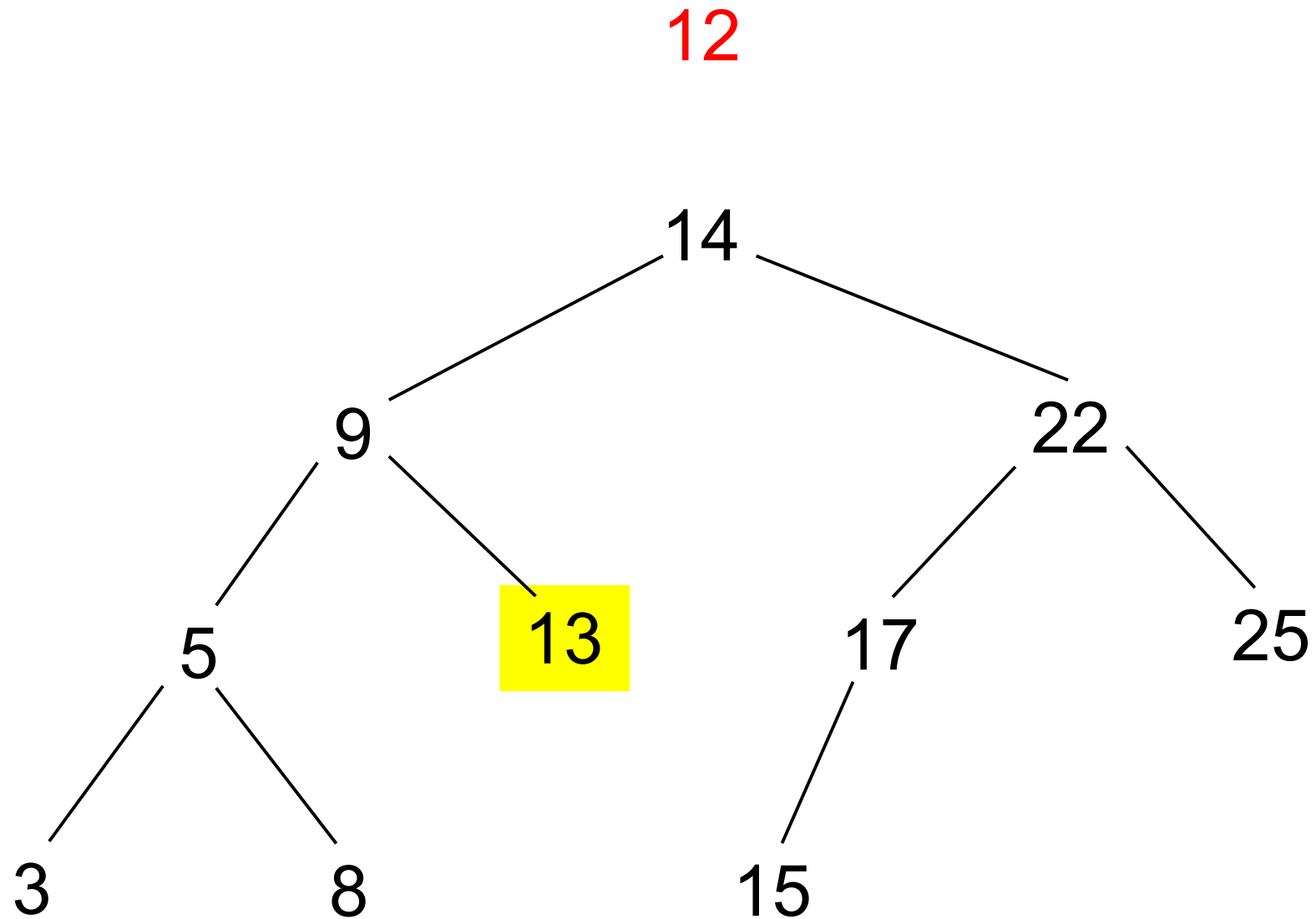


Inserting a New Element

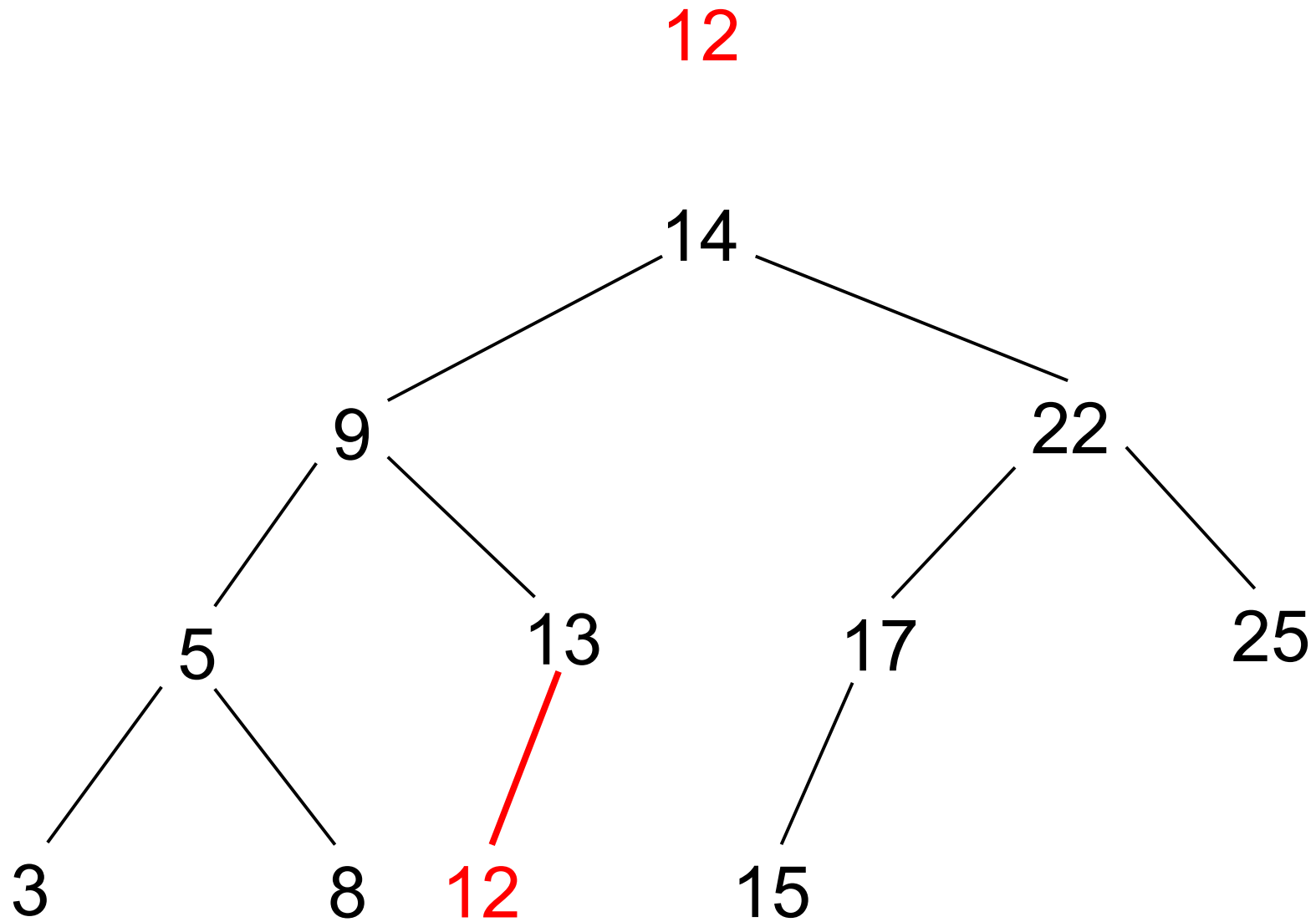
12



Inserting a New Element



Inserting a New Element



Inserting a New Element

$O(\log n)$ time complexity

