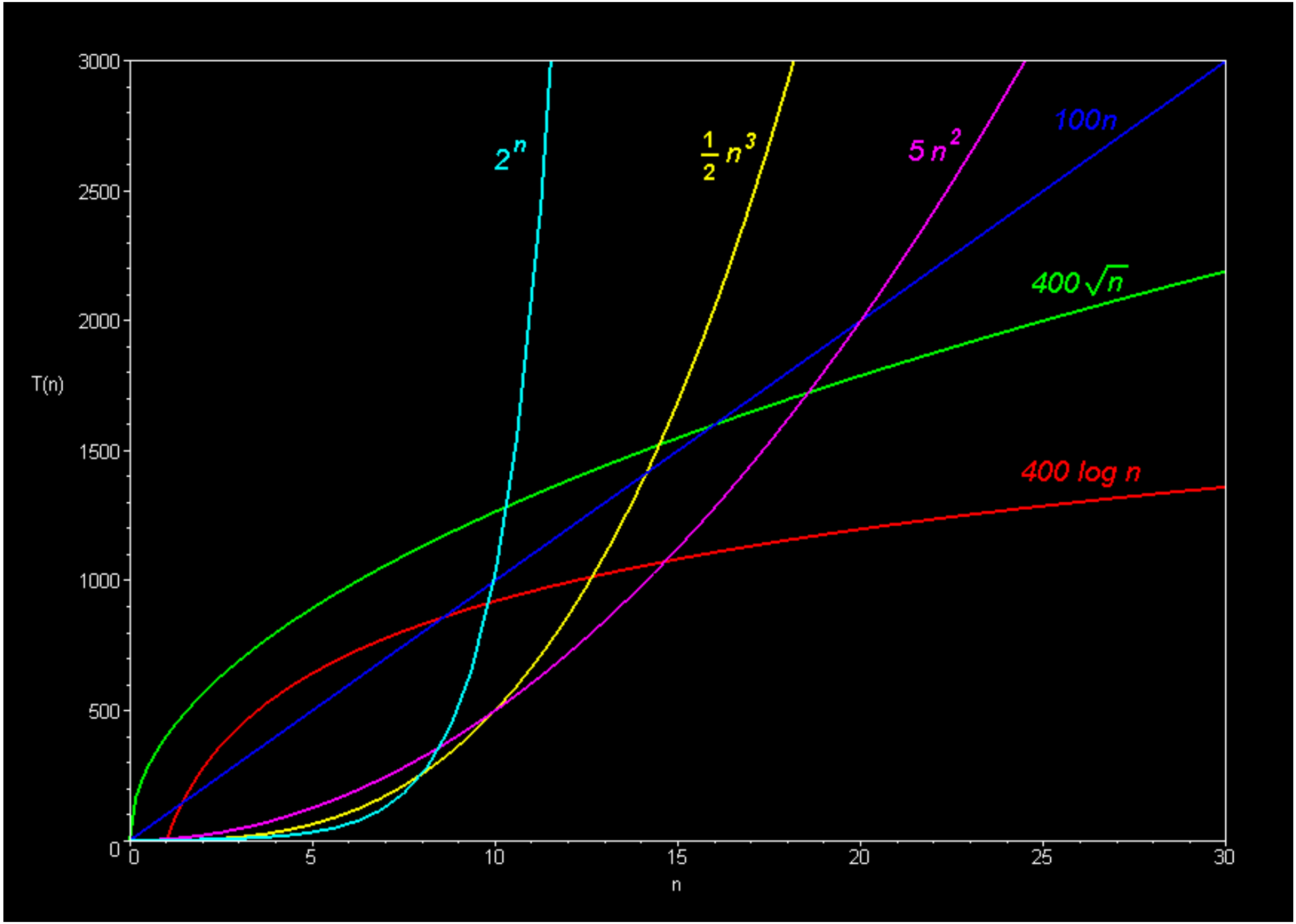
Comparing the Time Complexity of Algorithms

- Linear: $O(n)$ $\mathrm{T}(n) = 100n$
- Quadratic: $O(n^2)$ $\mathrm{T}(n) = 5n^2$
- Cubic: $O(n^3)$ $\mathrm{T}(n) = \frac{1}{2}\, n^3$
- Logarithmic: $O(\log n)$ $\mathrm{T}(n) = 400 \log n$
- Exponential: $O(2^n)$ $\mathrm{T}(n) = 2^n$
- Square root: $O(\sqrt{n})$ $\mathrm{T}(n) = 400 \sqrt{n}$

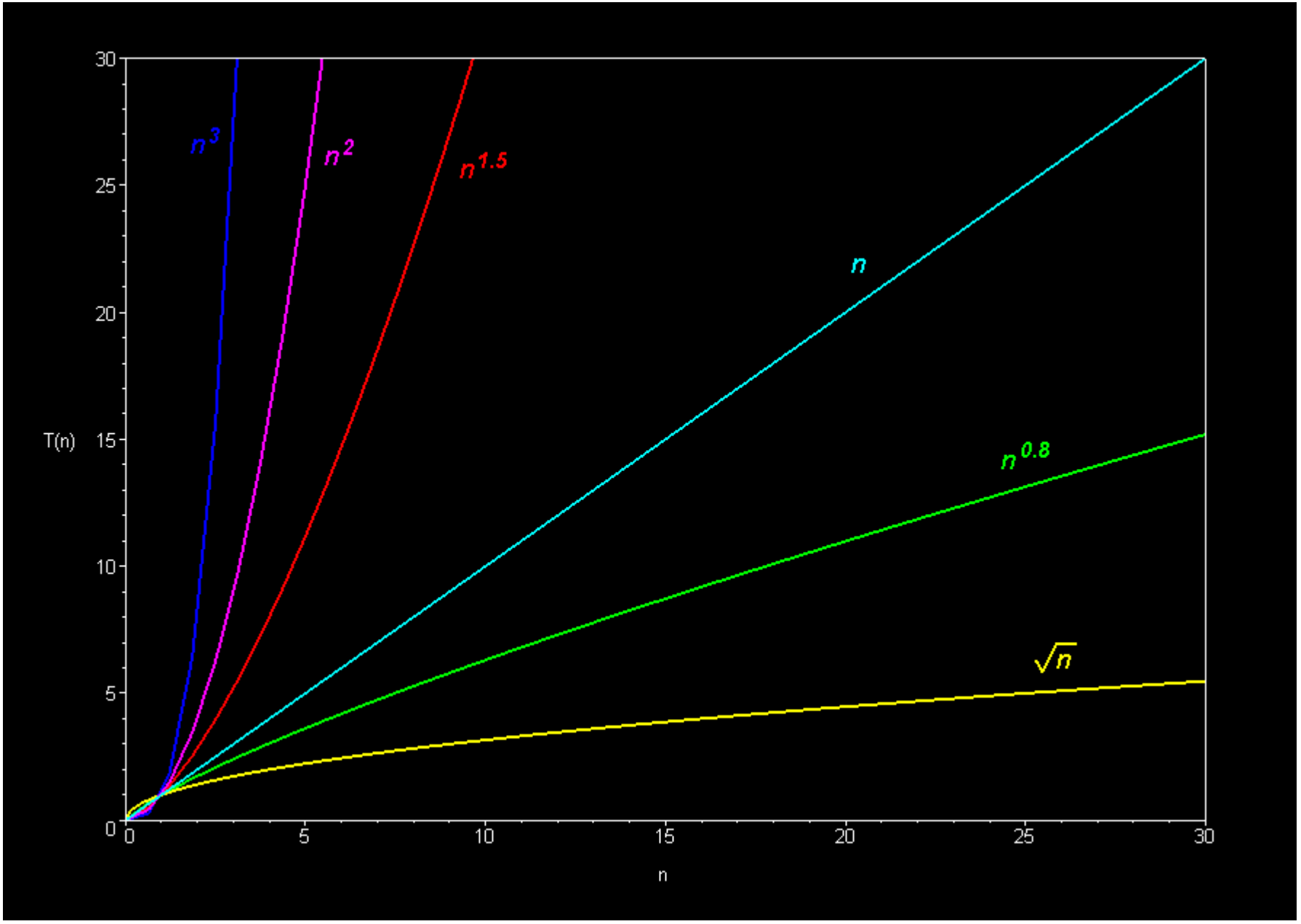

3000
2500
2000
1500
1000
500
0
T(n)
0
5
10
15
20
25
30
n
2^n
$\frac{1}{2}n^3$
$5n^2$
$100n$
$400\sqrt{n}$
$400 \log n$

Comparison of algorithms in terms of the maximum problem size they can handle:

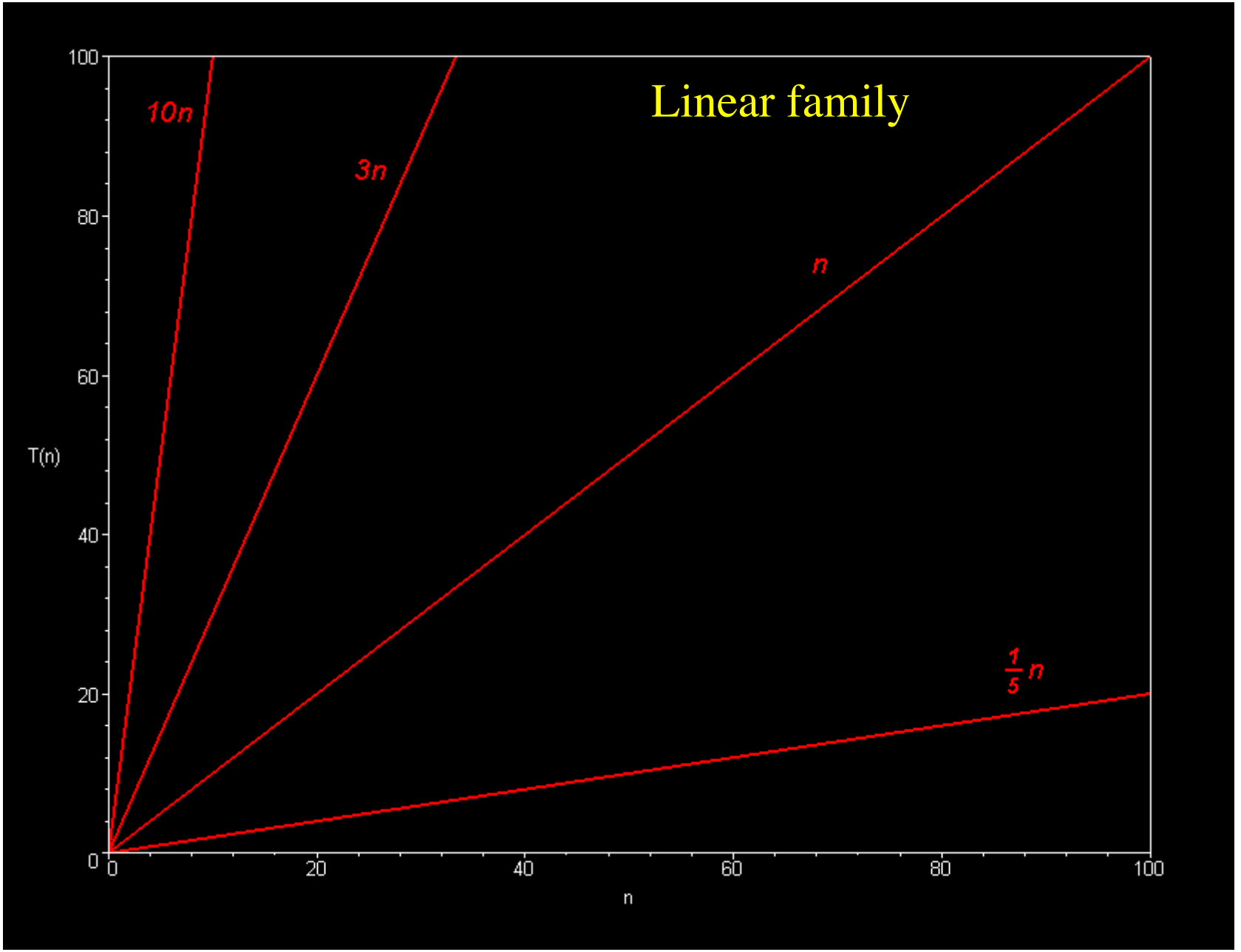
Algorithm Complexity	Running time T(n) (measured in seconds on an Apple Delicious computer)	Maximum problem size given 1000 seconds on an Apple Delicious	Computer Speed x 10 Maximum problem size given 1000 seconds on a Power Delicious (or 10,000 seconds on a classic Delicious)
O(n)	100n	n = 10	n = 100 *(x 10 increase)*
$O(n^2)$	$5n^2$	n = 14	n = 45 *(x 3)*
$O(n^3)$	$\frac{1}{2} n^3$	n = 12	n = 27 *(x 2)*
$O(2^n)$	2^n	n = 10	n = 13 *(x 1.3)*
O(sqrt n)	400 sqrt(n)	n = 6	n = 625 *(x 100)*
O(log n)	400 log(n)	n = 12	n = 72 billion *(x 6 billion)*

MORAL: Cheaper, faster computers mean bigger problems to solve. Bigger problems to solve mean efficiency is *more* important, not less!

The basic shape of a polynomial function is determined by the highest valued exponent in the polynomial (called the *order* of the polynomial).



Multiplicative constants do not affect the fundamental shape of a curve. Only the steepness of the curve is affected.



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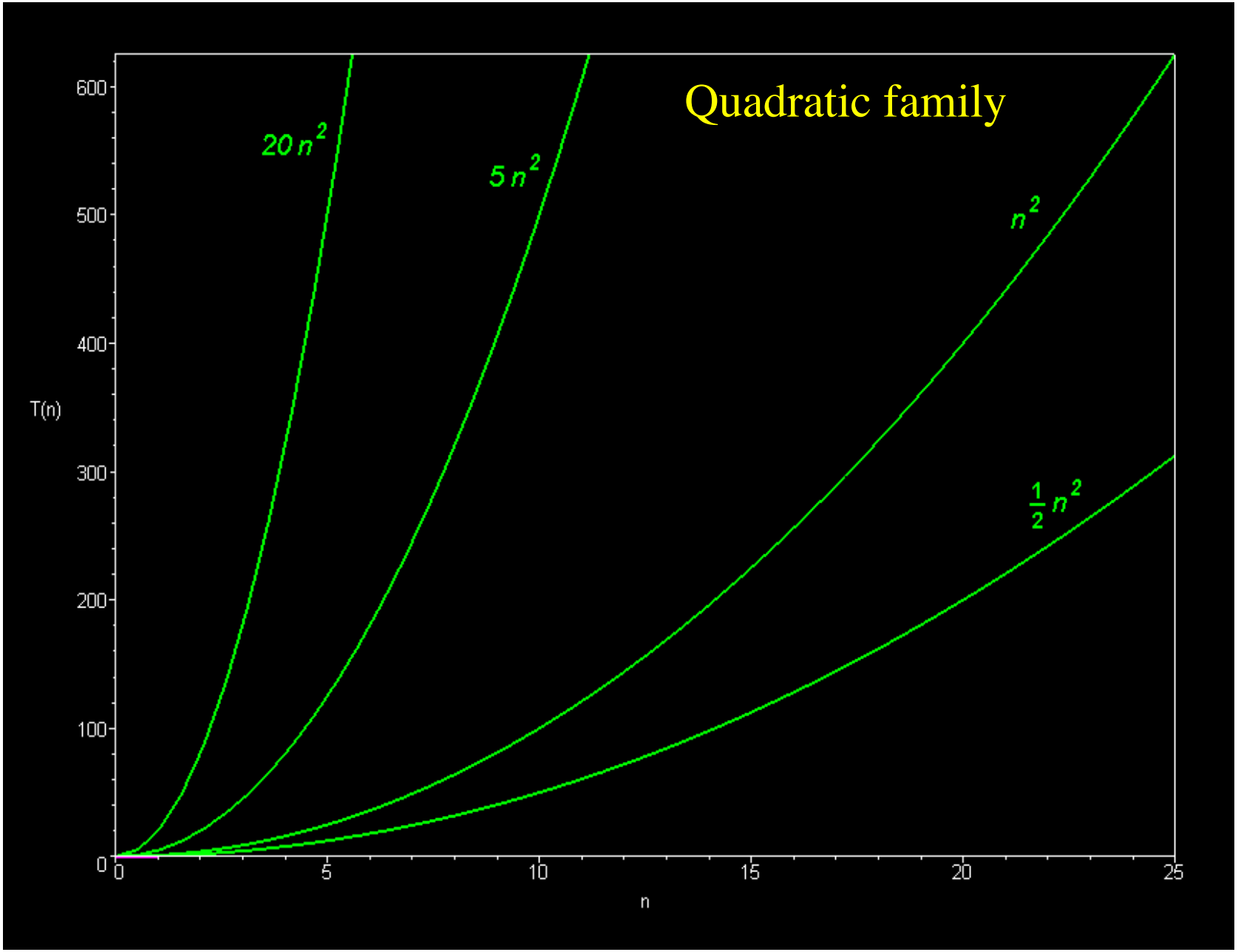

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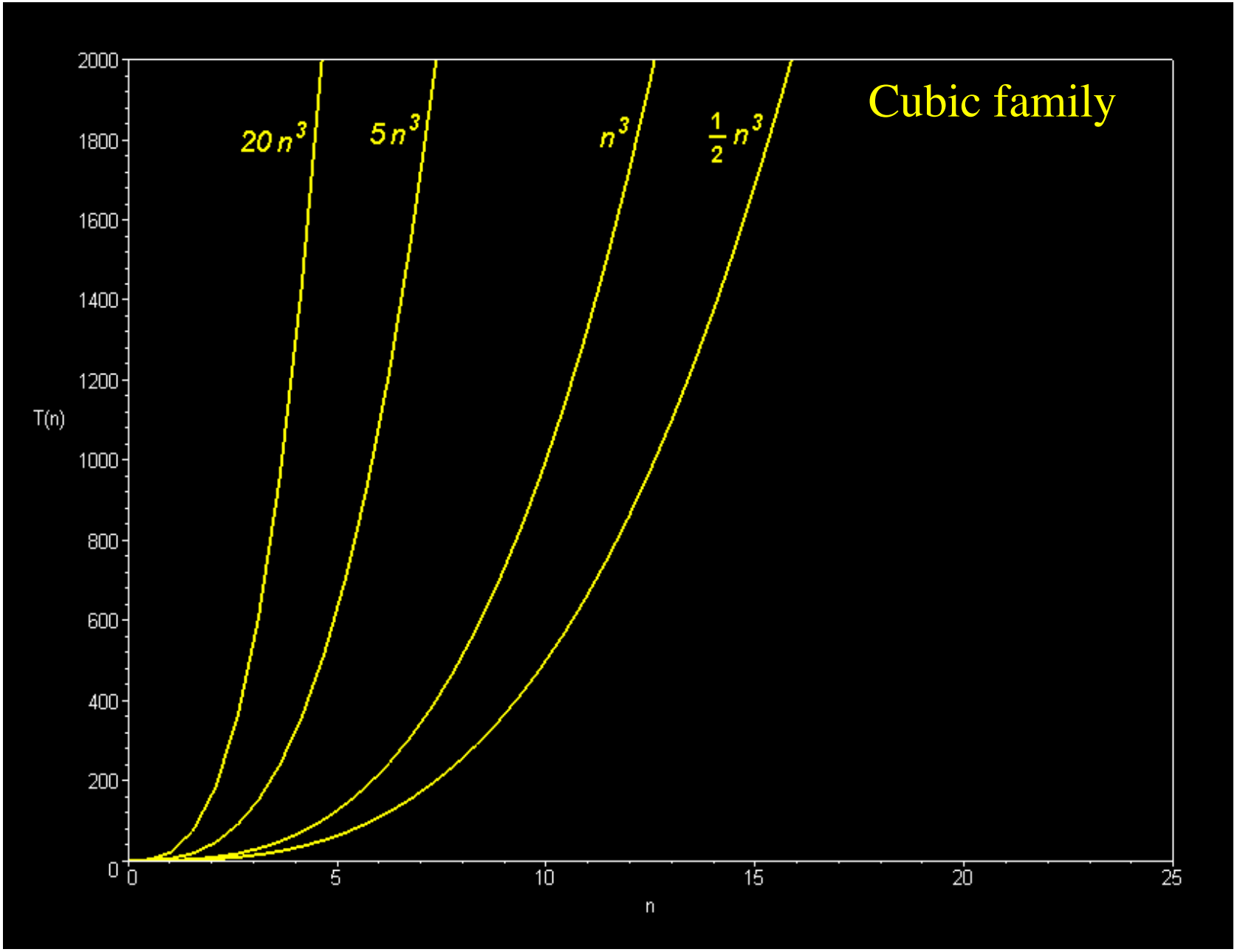
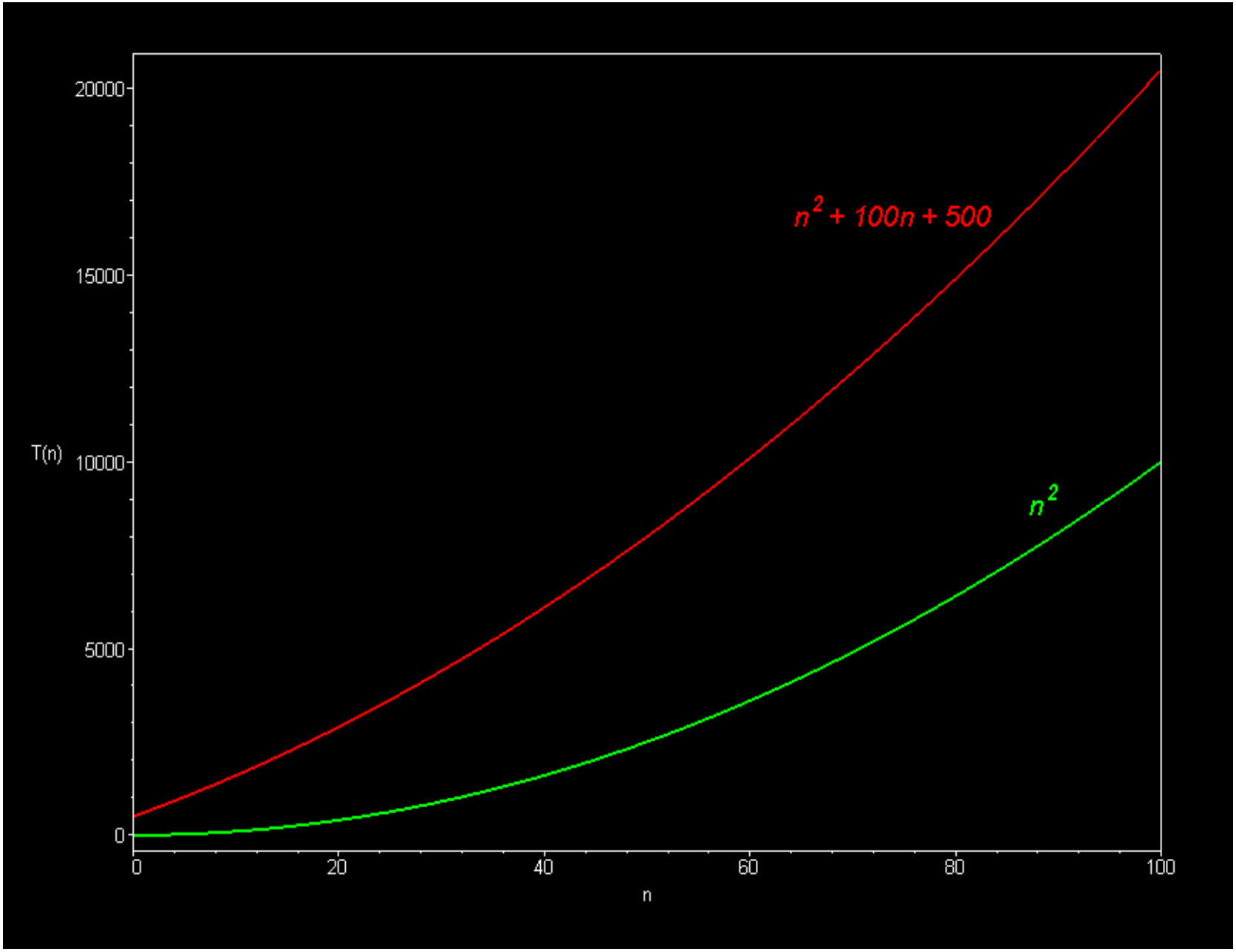


Only the *dominant terms* of a polynomial matter in the long run. Lower-order terms fade to insignificance as the problem size increases.



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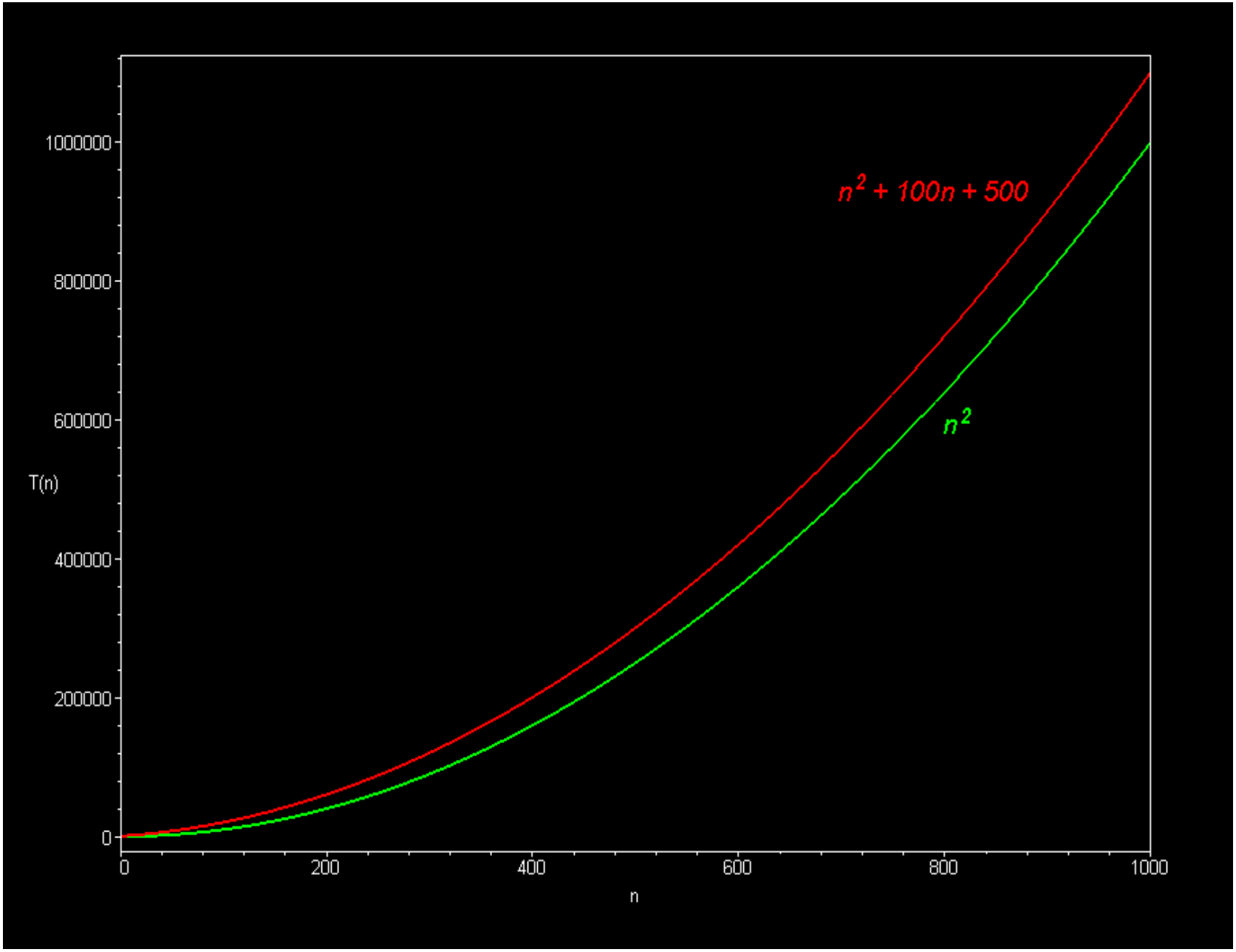

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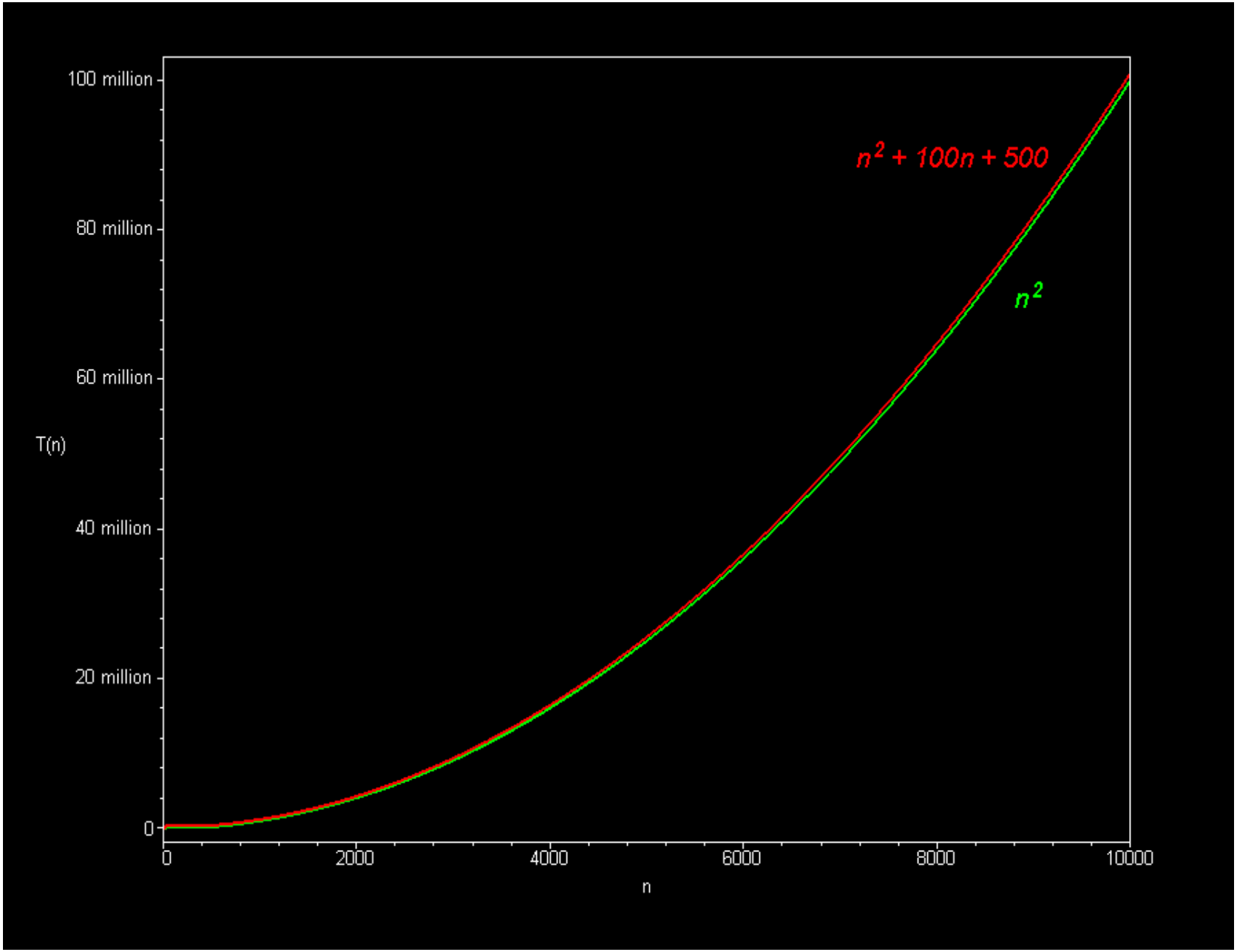
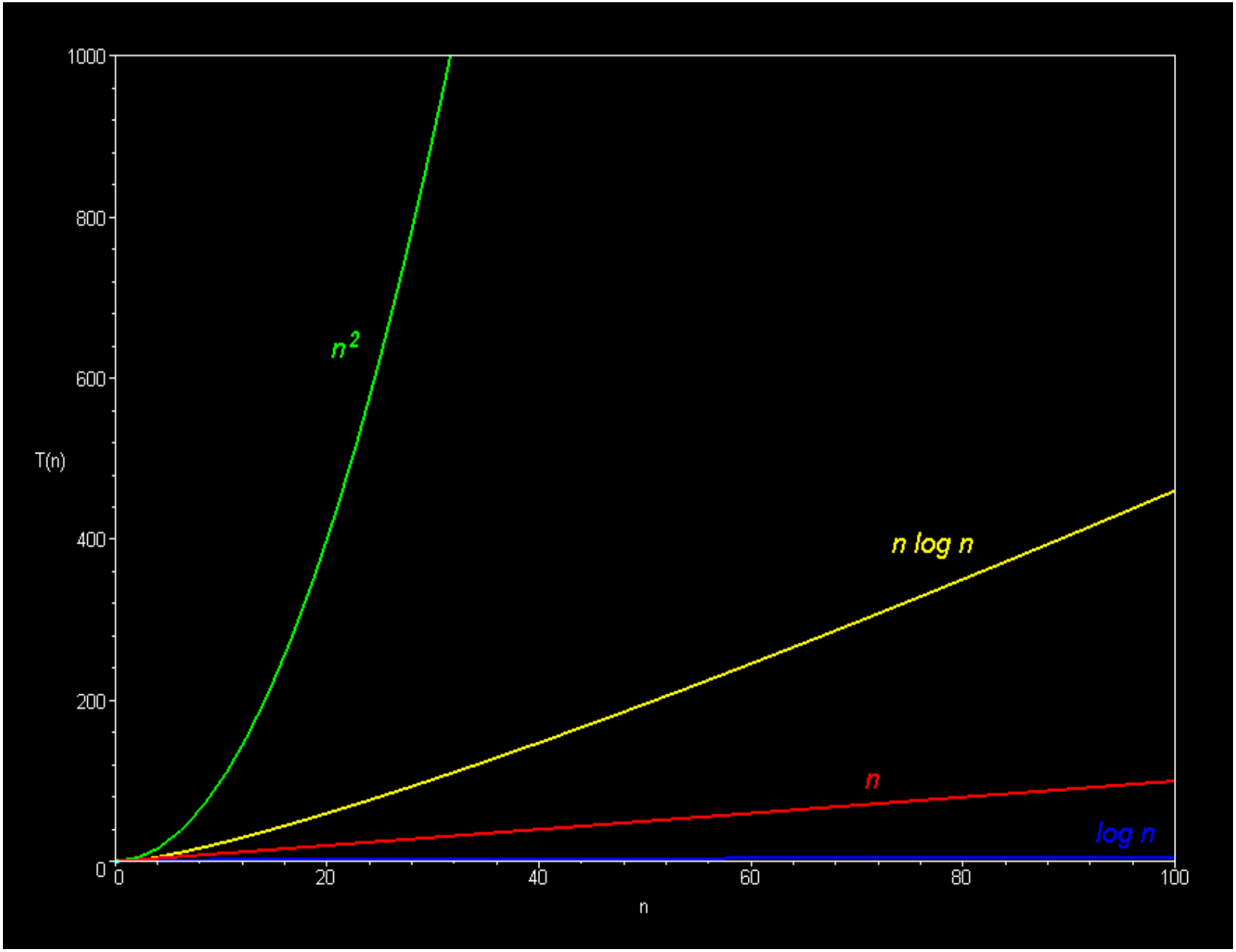
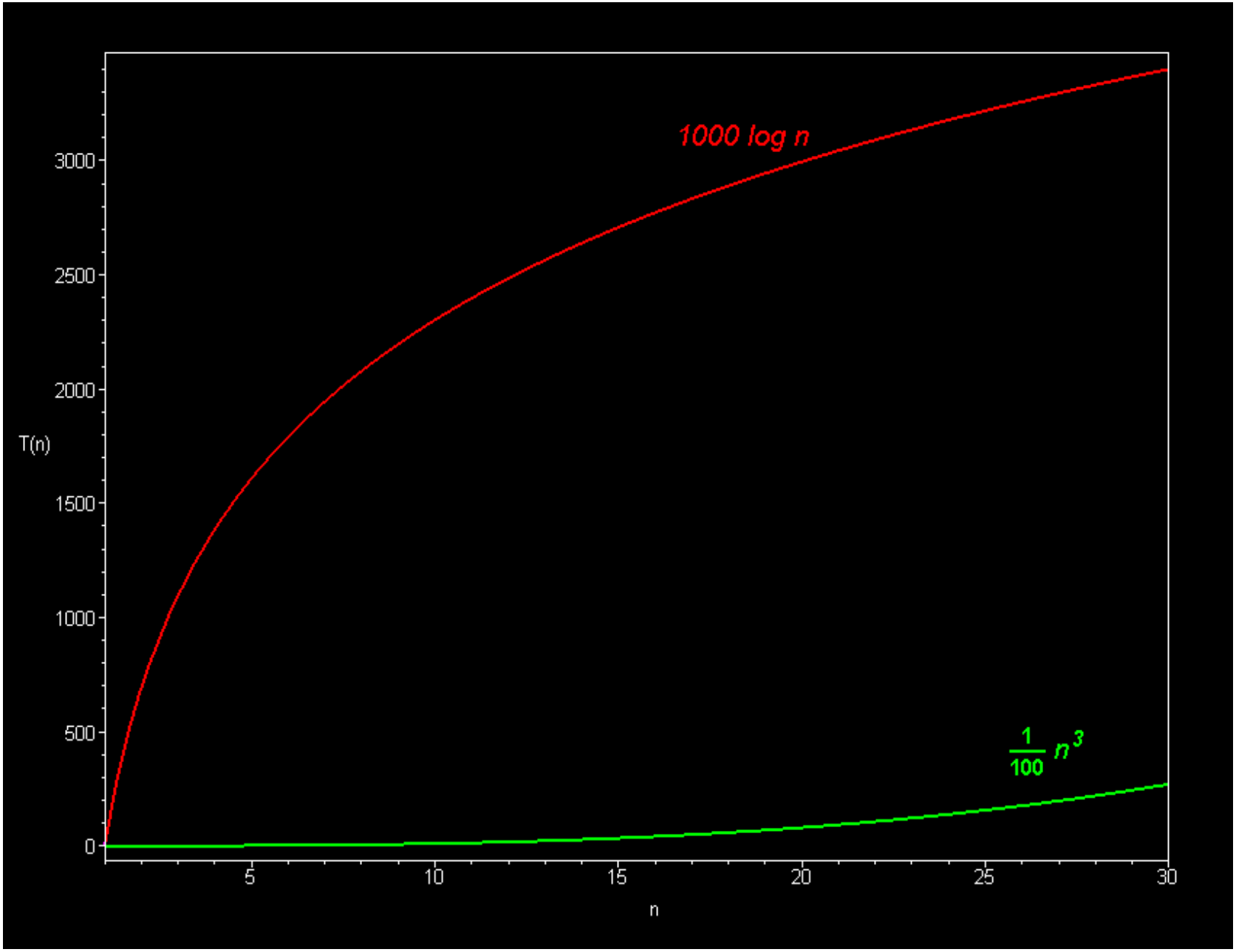


The best sorting algorithms (e.g., mergesort) run in O(n log n) time. Slower ones (bubble sort, selection sort, insertion sort), take O(n^2) time.



Polynomial curves will always overtake logarithmic curves eventually, when the problem size gets big enough, regardless of the multiplicative constants.



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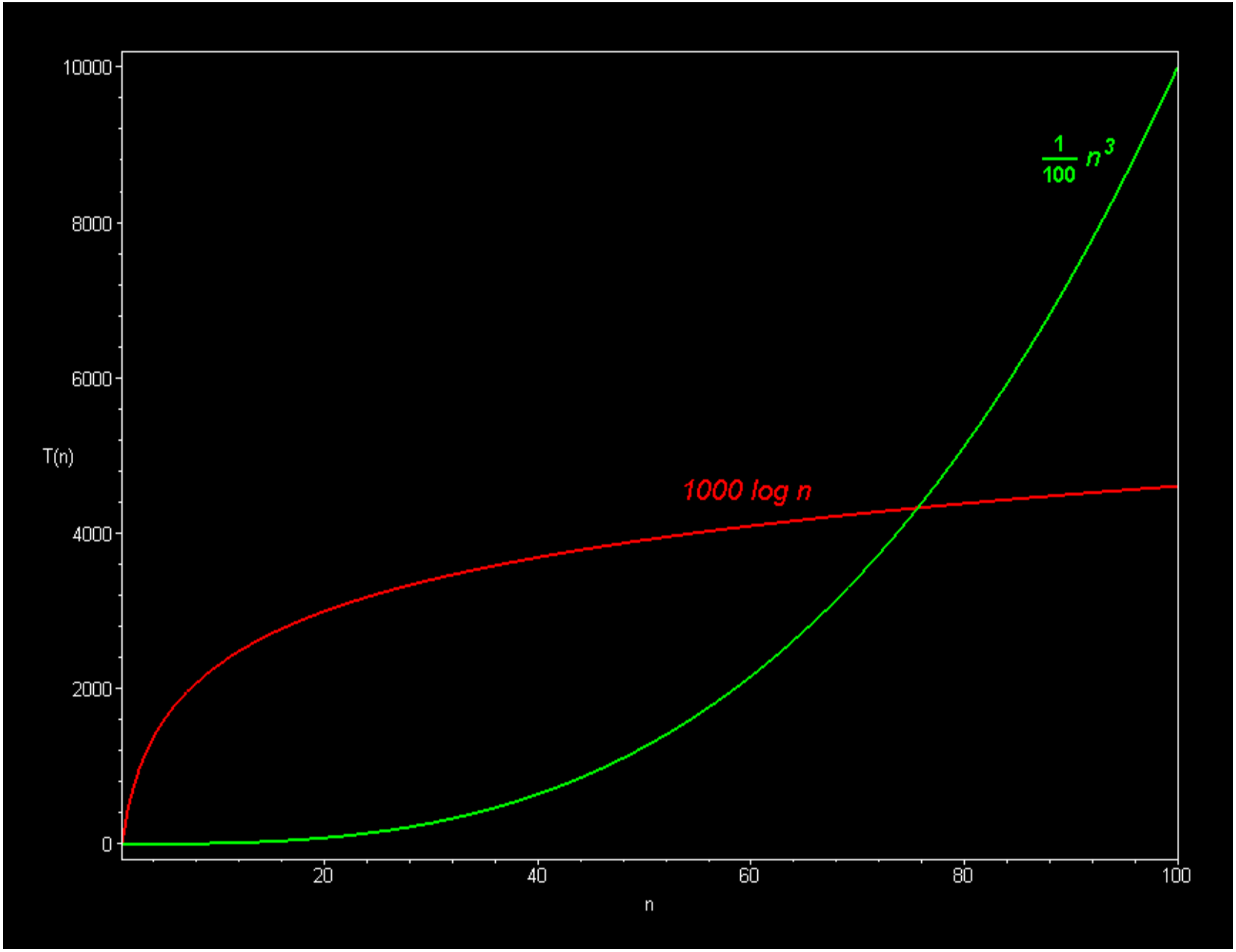
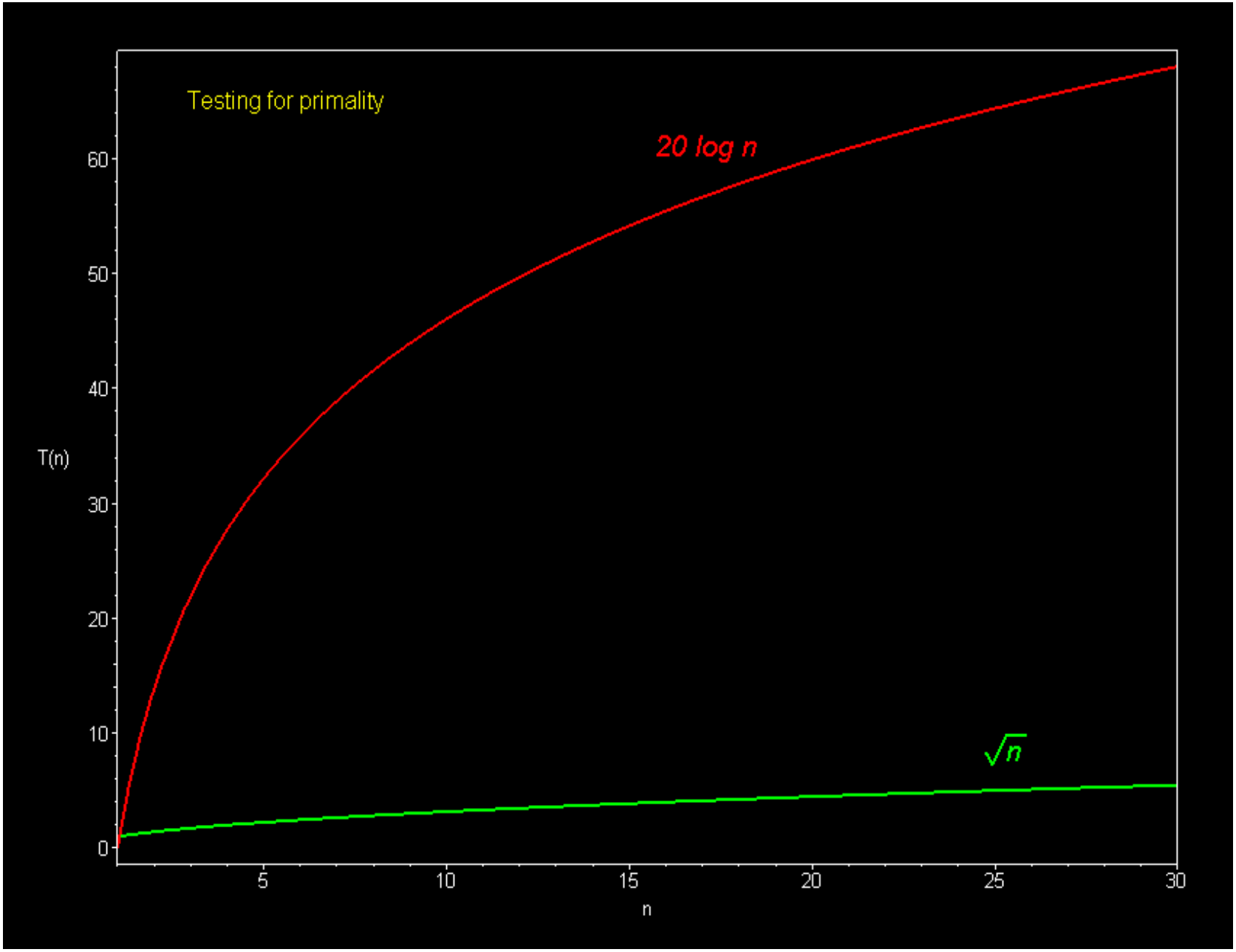


The superiority of the O(log *n*) Fermat prime test over the O(sqrt *n*) prime test becomes clear for really big integers.



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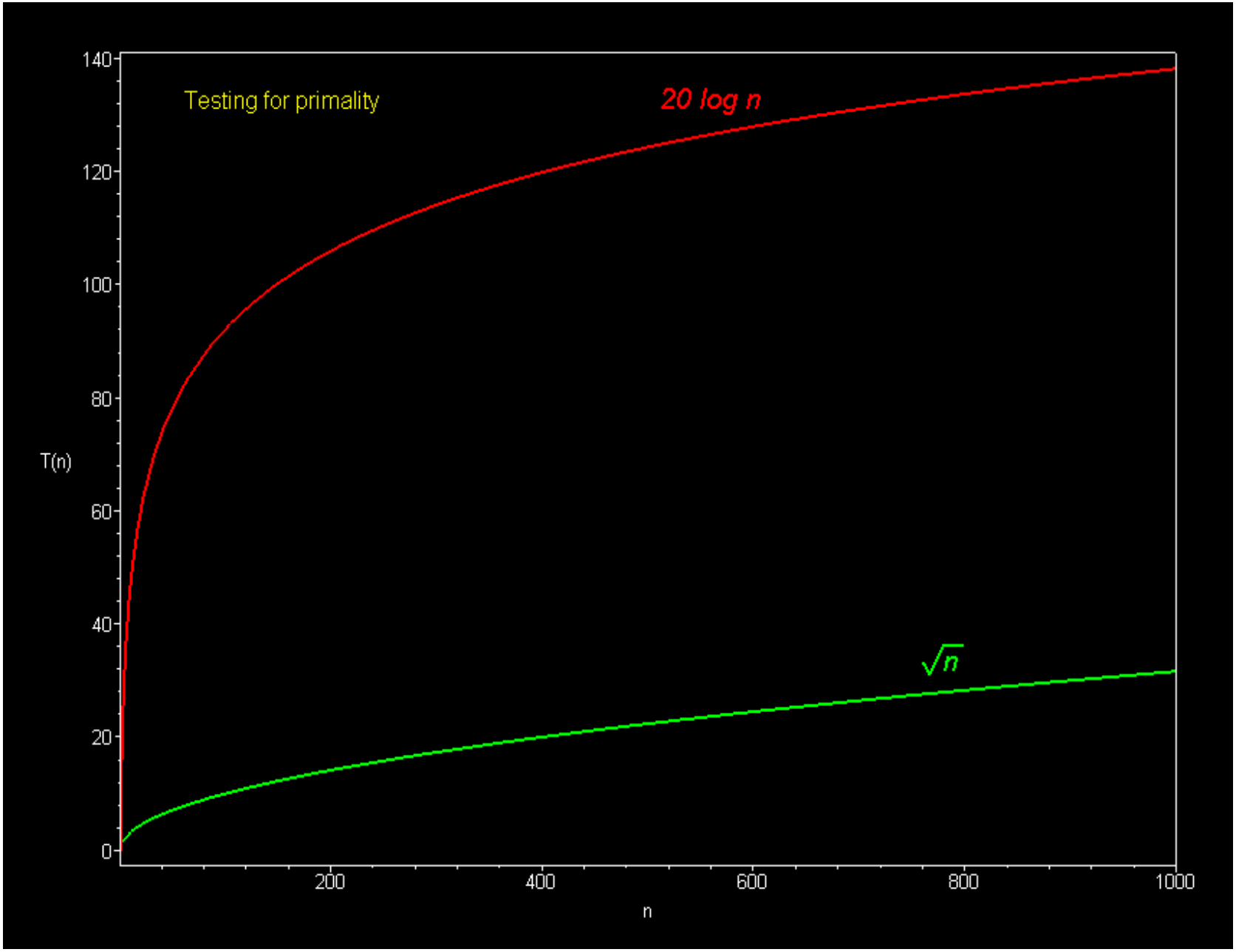

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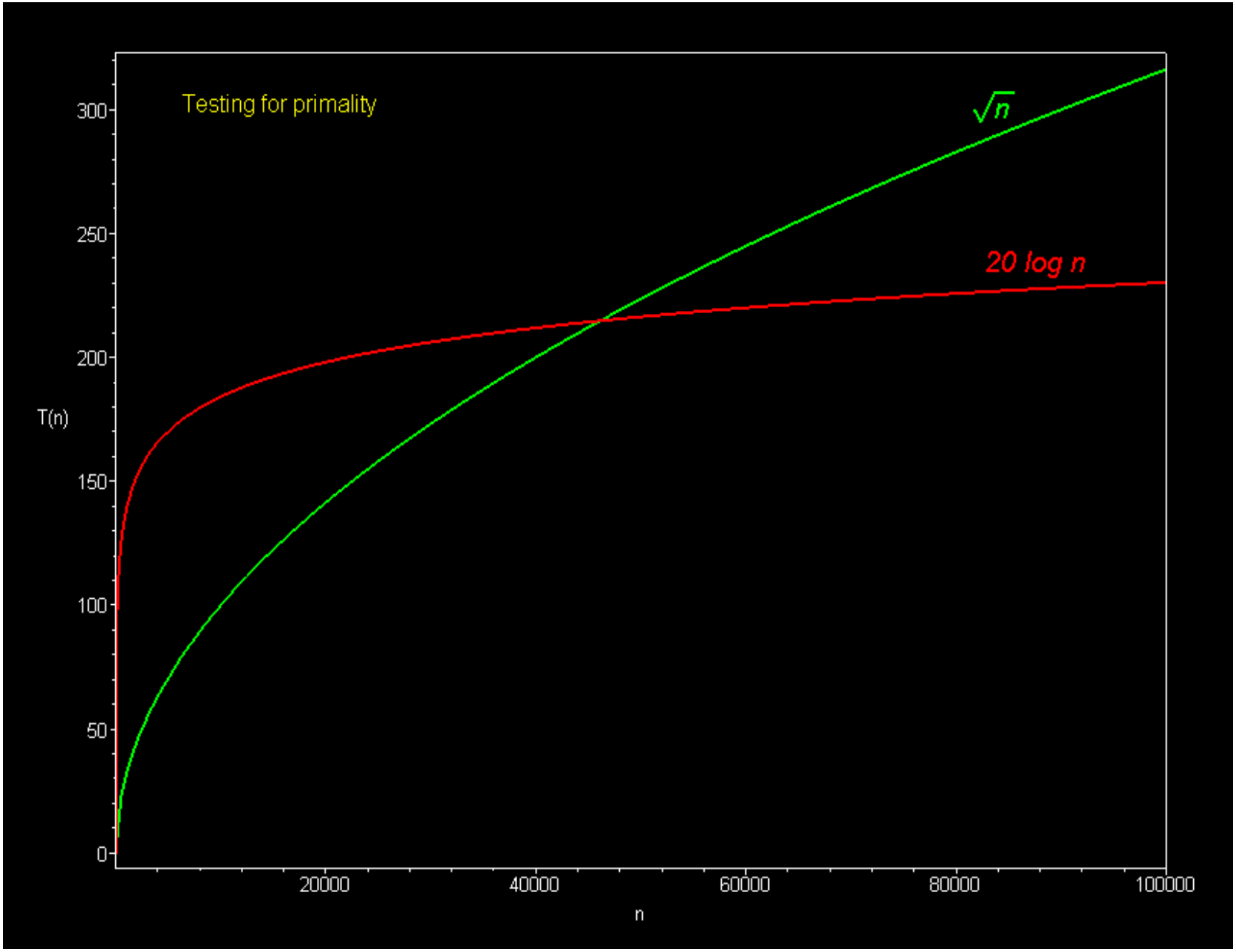

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