

# Introduction to Recursive Thinking

A giant pile of numbers!



# How to add up all the numbers in the pile?

1. set *running\_total* equal to 0
2. take a number out of the pile (call it  $N$ )
3. add  $N$  to *running\_total*
4. if there are still numbers left in the pile, go back to step 2
5. otherwise, report the value of *running\_total* as your answer



*If there are **millions of numbers** in the pile,  
you will need to perform **millions of steps***

*(Way too much busy work!)*

How to add up all the numbers in the pile?

*Instead, we are going to use **magic!***

# How to add up all the numbers in the pile?

1. if the pile is empty, report 0 and stop
2. otherwise, **ask a wizard** to add up the whole pile for you
3. report whatever value the wizard told you as your answer



# How to add up all the numbers in the pile?

1. if th

2. oth

3. rep

*“Don't be such a slacker!  
I'll help you out, but you have to do  
at least a little bit of the work yourself!”*



# How to add up all the numbers in the pile?

1. if the pile is empty, report 0 and stop
2. otherwise, take a number out of the pile (call it  $N$ )
3. **ask the wizard** to add up the remaining numbers in the pile for you
4. add  $N$  to whatever the wizard told you, and report that as your answer



*Even if there are **millions of numbers** in the pile, **you still need to perform only a few steps***



# How to find the largest number in the pile?

1. if the pile is empty, there is no answer
2. if the pile contains just one number, report that as your answer, and stop
3. otherwise, take a number out of the pile (call it  $N$ )
4. **ask the wizard** to find the largest number in the remaining pile
5. compare  $N$  to what the wizard told you, and report the larger value as your answer





# How to count all the numbers in the pile?

1. if the pile is empty, report 0 and stop
2. otherwise, take a number out of the pile
3. **ask the wizard** to count how many numbers are in the remaining pile
4. add 1 to whatever the wizard told you, and report that as your answer



# How to check if a specific number is in the pile?

1. if the pile is empty, report no and stop
2. otherwise, take a number out of the pile
3. if this is the number you're looking for, report yes and stop
4. otherwise, **ask the wizard** whether the number you're looking for is in the remaining pile
5. report as your answer whatever the wizard told you



# How to check if the numbers are all the same?

1. if the pile is empty, there is no answer
2. if the pile contains just one number, report yes and stop
3. otherwise, take a number out of the pile (call it  $N$ )
4. **ask the wizard** if the pile of remaining numbers are all the same
5. if the wizard said no, report no as your answer and stop
6. if the wizard said yes, check whether  $N$  is the same as the remaining numbers
7. if so, report yes, otherwise report no



# How to create a new pile containing the *squares* of all the numbers in the original pile?

1. if the pile is empty, just create a new empty pile, and stop
2. otherwise, take a number out of the pile (call it  $N$ )
3. **ask the wizard** to give you the squares of all the numbers in the remaining pile
4. put  $N^2$  into the pile of squares the wizard gave you, and give back this new pile as your answer



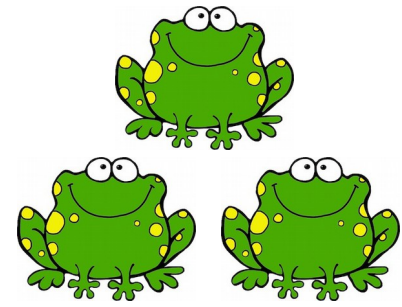
# How to create a new pile containing just the *odd numbers* from the original pile?

1. if the pile is empty, just create a new empty pile, and stop
2. otherwise, take a number out of the pile (call it  $N$ )
3. **ask the wizard** to give you just the odd numbers from the remaining pile
4. if  $N$  is not odd, just give back the pile the wizard gave you as your answer
5. otherwise, put  $N$  into the pile the wizard gave you and give back this new pile as your answer



1  
2 3

# How to turn a pile of numbers into a pile of frogs?

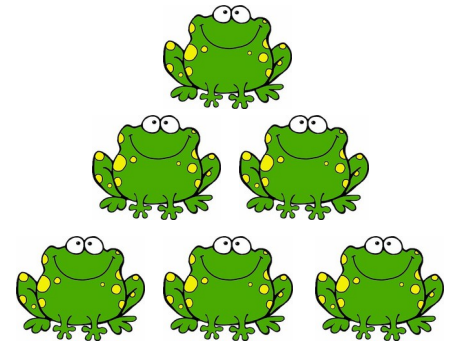


1. if the pile of numbers is empty, just create a new empty pile of frogs, and stop
2. otherwise, take a number out of the pile and turn it into a frog
3. **ask the wizard** to turn the remaining pile of numbers into frogs
4. add your frog to the wizard's pile and give back this new pile as your answer



# How to create a brand new pile of $N$ frogs?

1. if the requested number of frogs is 0, just create an empty pile of frogs, and stop
2. otherwise, **ask the wizard** to create a pile containing  $N - 1$  frogs
3. add one more frog to the wizard's pile



# How to calculate the factorial of $N$ ?

(“factorial of  $N$ ” is written “ $N!$ ”)

$$0! = 1 \quad (\text{by definition})$$

$$N! = N \times N-1 \times N-2 \times \dots \times 3 \times 2 \times 1$$

*Let the wizard do this part for you*

$$\begin{aligned} \text{Example: } 5! &= 5 \times 4 \times 3 \times 2 \times 1 \\ &= 120 \end{aligned}$$



# Summary: Solving a Problem Recursively

## 1. Base Case:

solve the simplest version(s) of the problem directly

## 2. General Case:

(a) make the problem slightly smaller

(b) let the recursion “magically” solve the smaller version of the problem for you

(c) use the result of (b) to help you solve the original version of the problem

